



# Combining 3D Shape, Color, and Motion for Robust Anytime Tracking

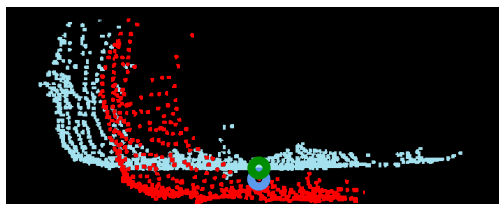
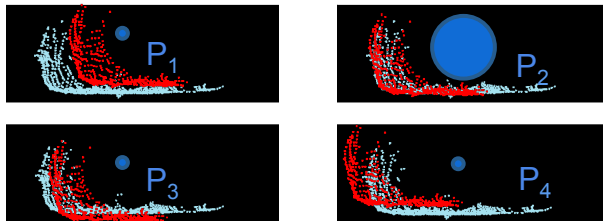
David Held, Jesse Levinson, Sebastian Thrun, and Silvio Savarese

Stanford  
University

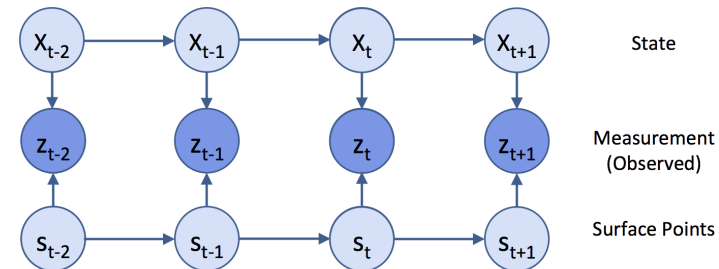
## Goal: Fast and Robust Velocity Estimation



## Our Approach: Alignment Probability

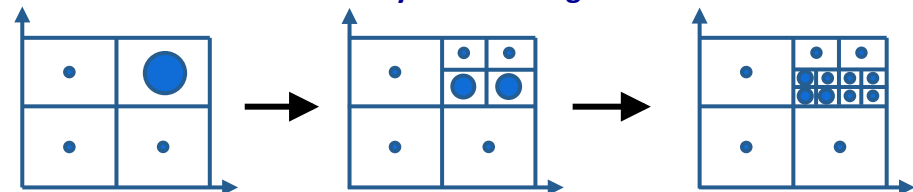


- Spatial Distance
- Color Distance (if available)
- Probability of Occlusion



$$p(x_t \mid z_1 \dots z_t) = \eta p(z_t \mid x_t, z_{t-1}) p(x_t \mid z_1 \dots z_{t-1})$$

## Annealed Dynamic Histograms



Raw Points

ADH Tracker

ICP Baseline



# Combining 3D Shape, Color, and Motion for Robust Anytime Tracking

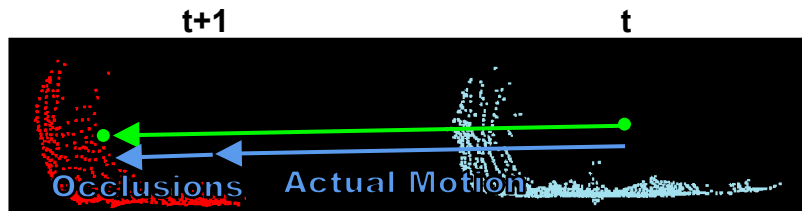
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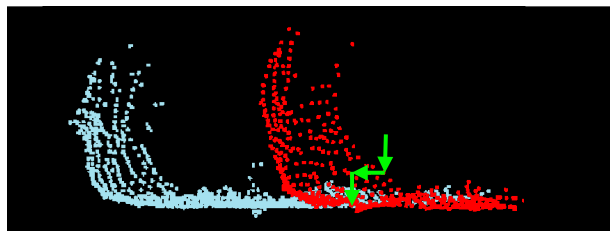
Goal: Fast and Robust Velocity Estimation



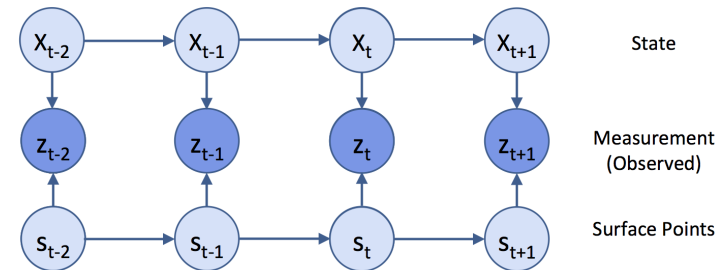
Baseline: Centroid Kalman Filter



Baseline: ICP

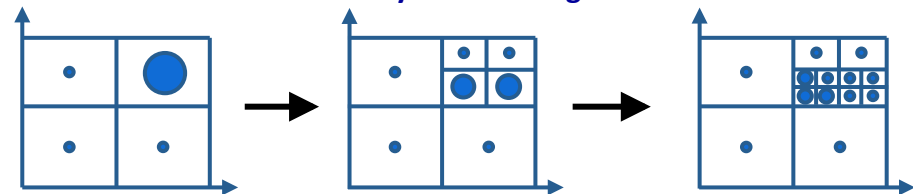


Local Search → Poor Local Optimum!



$$p(x_t \mid z_1 \dots z_t) = \eta p(z_t \mid x_t, z_{t-1}) p(x_t \mid z_1 \dots z_{t-1})$$

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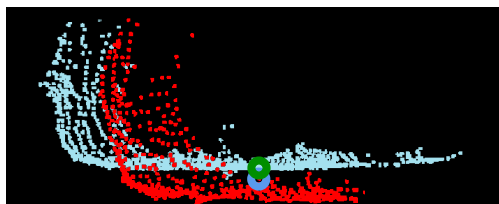
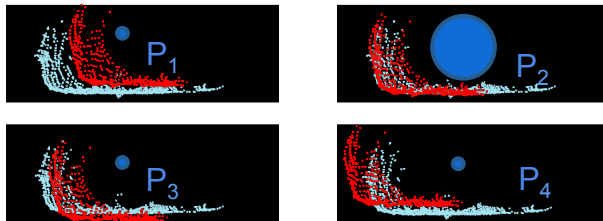
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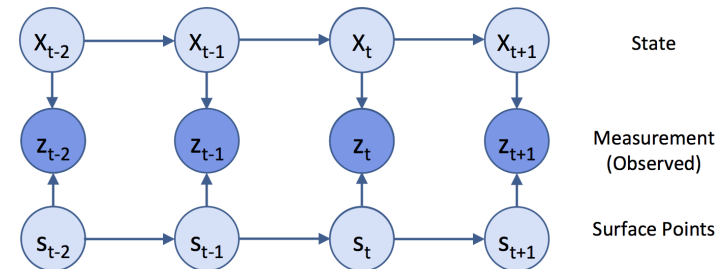
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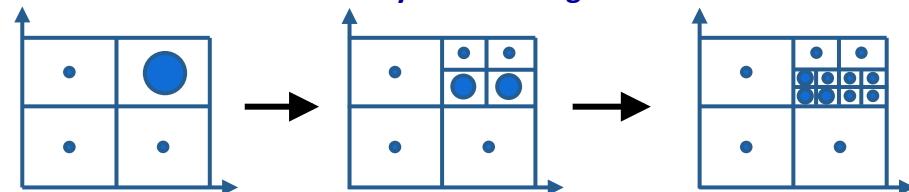


- Spatial Distance
- Color Distance (if available)
- Probability of Occlusion



$$p(x_t \mid z_1 \dots z_t) = \eta p(z_t \mid x_t, z_{t-1}) p(x_t \mid z_1 \dots z_{t-1})$$

## Annealed Dynamic Histograms



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# Motivation

Quickly and robustly estimate the speed of nearby objects

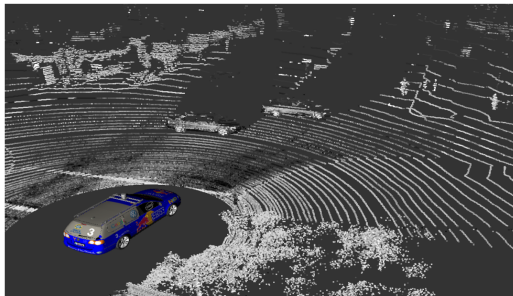


# System

**Camera Images**



**Laser Data**

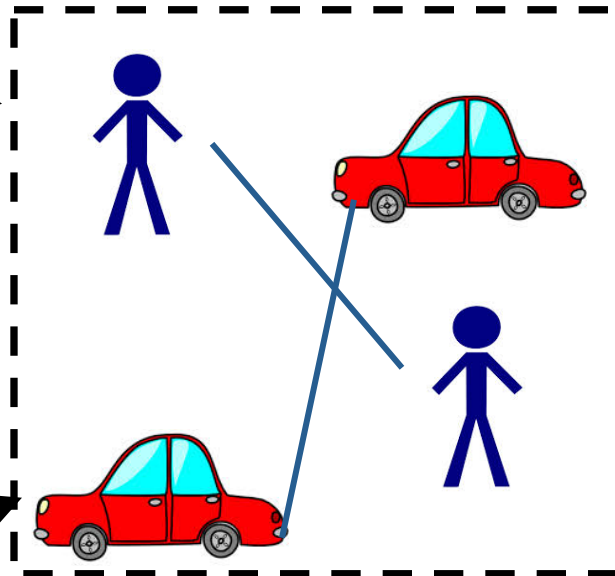
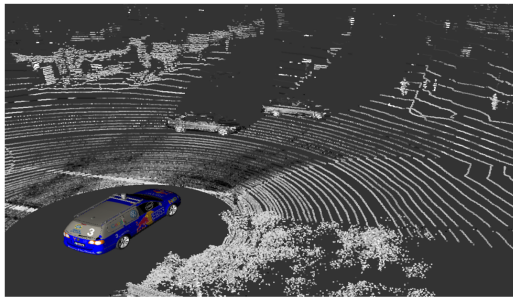


# System

Camera Images



Laser Data



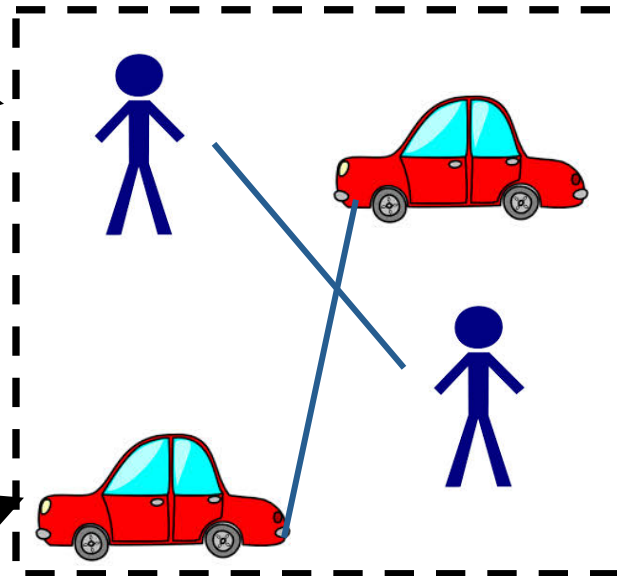
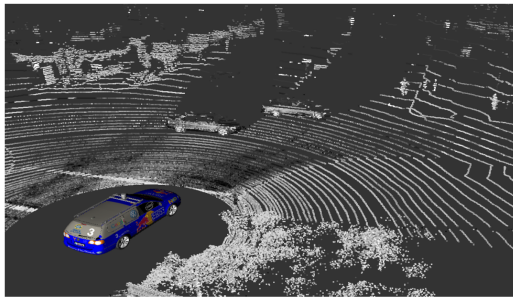
**Previous Work**  
(Teichman, et al)

# System

Camera Images



Laser Data



**Previous Work**  
(Teichman, et al)

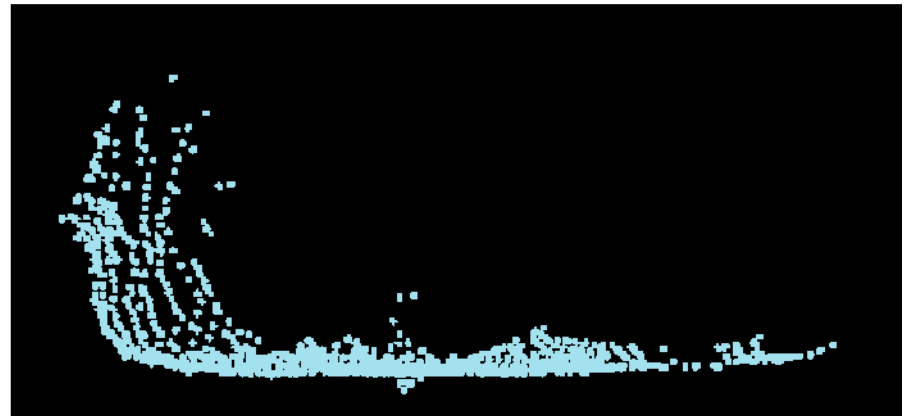
**Velocity  
Estimation**

**This Work**



# Velocity Estimation

**t**

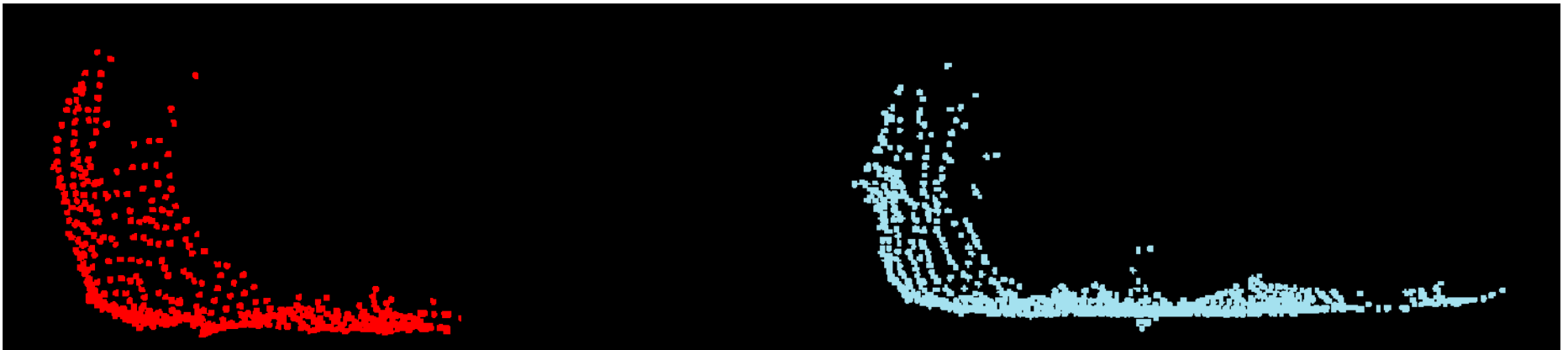




# Velocity Estimation

$t+1$

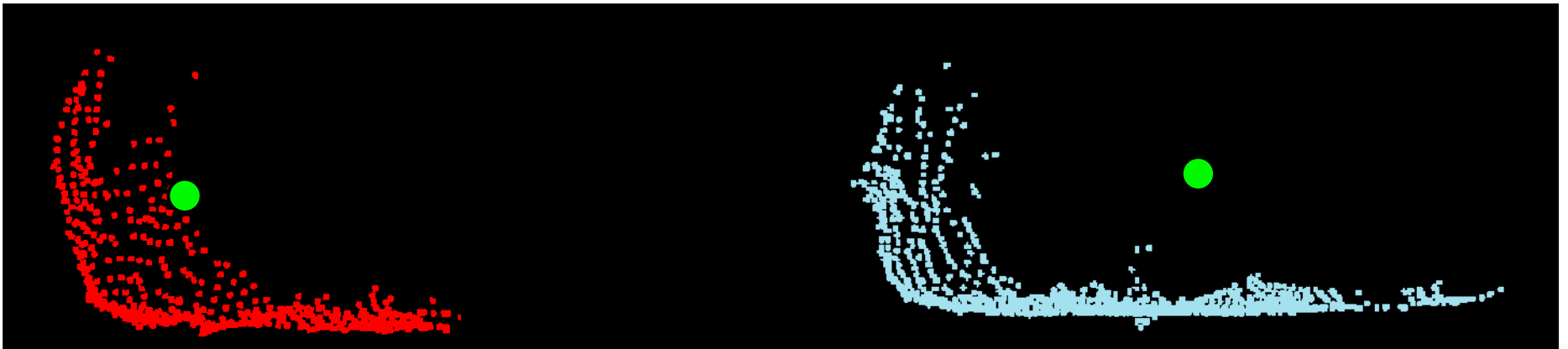
$t$



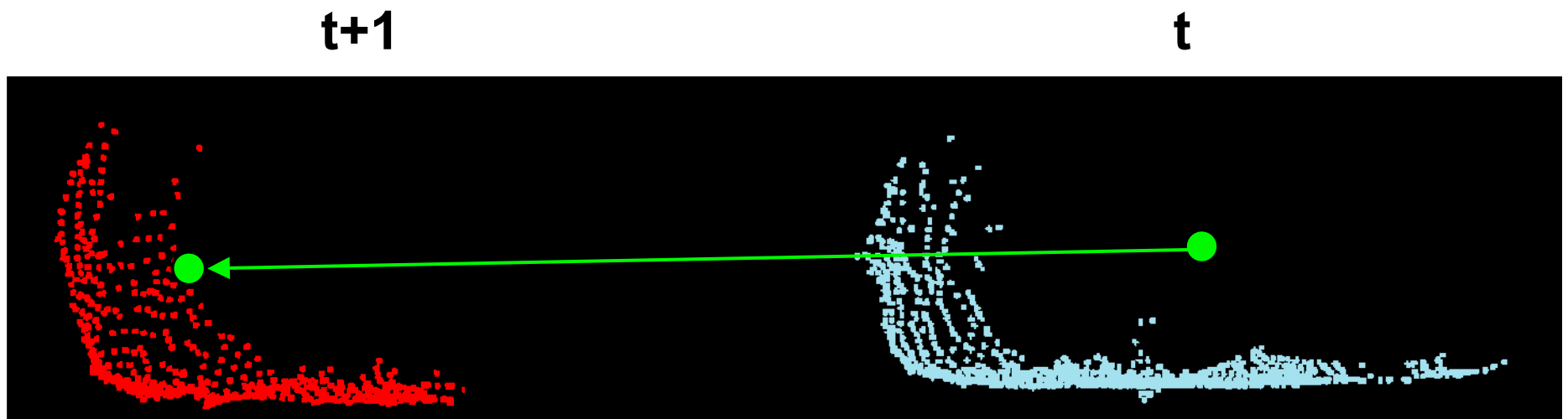
# Velocity Estimation

$t+1$

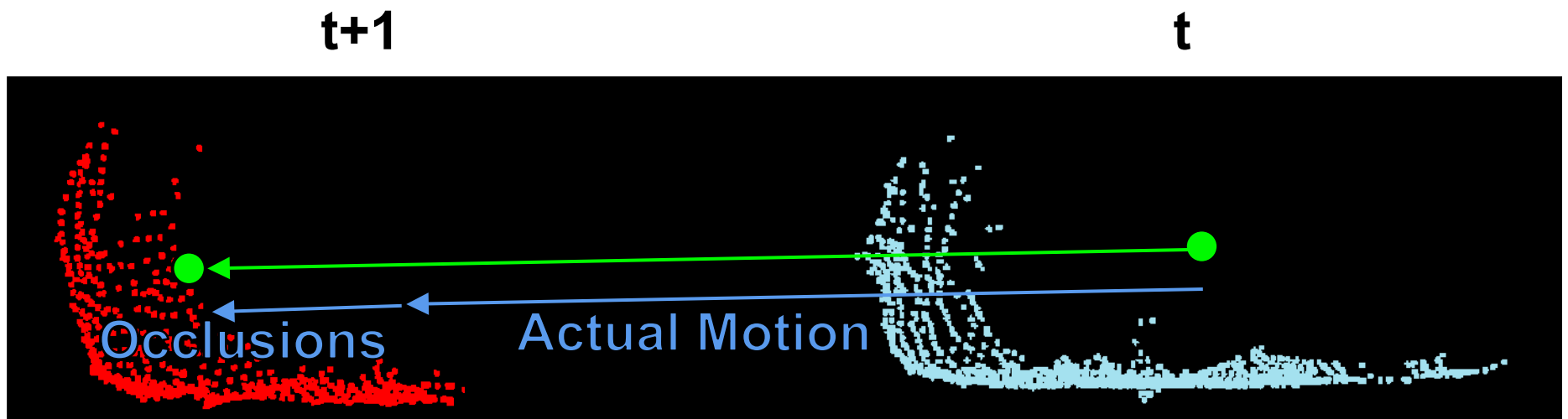
$t$



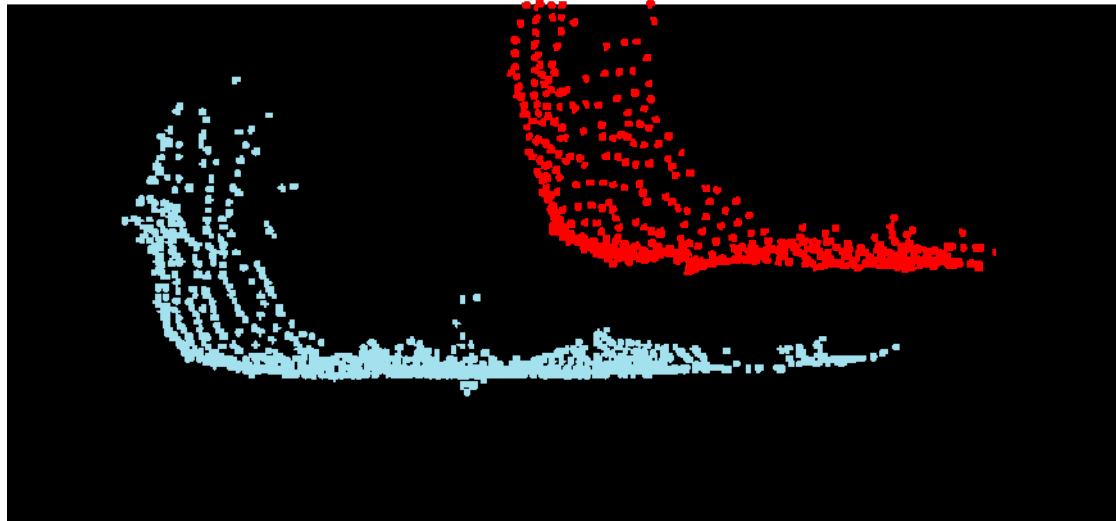
# Velocity Estimation



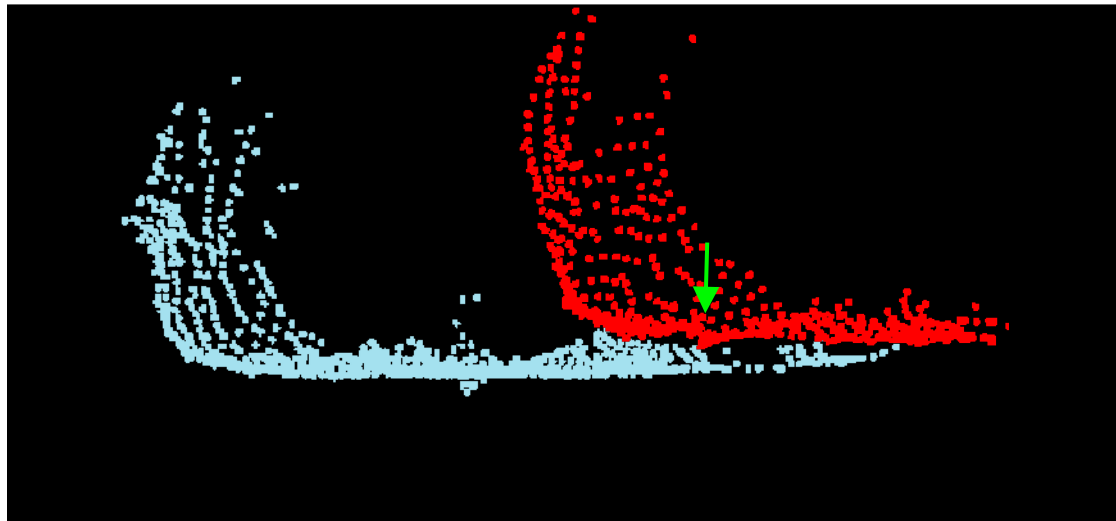
# Velocity Estimation



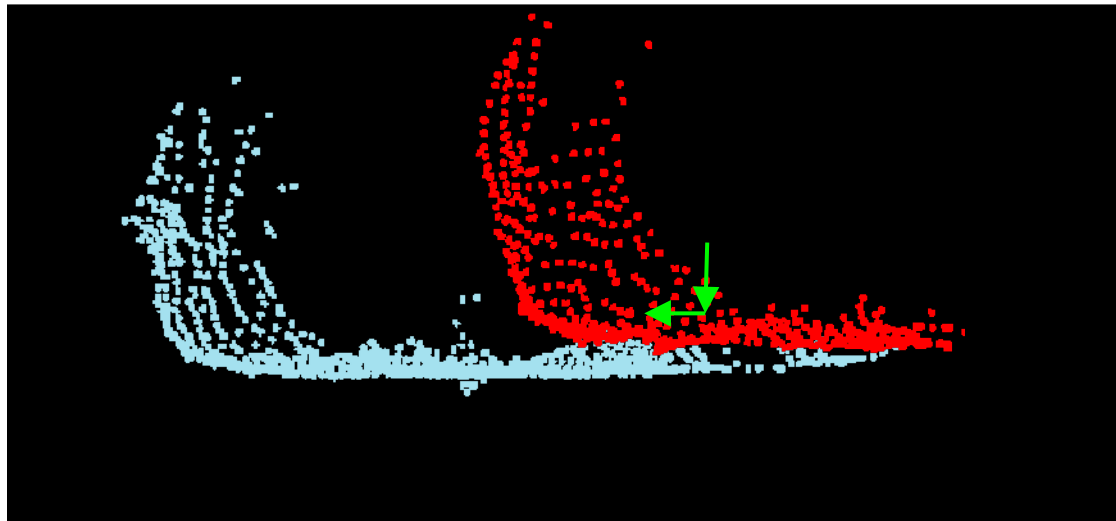
# ICP Baseline



# ICP Baseline

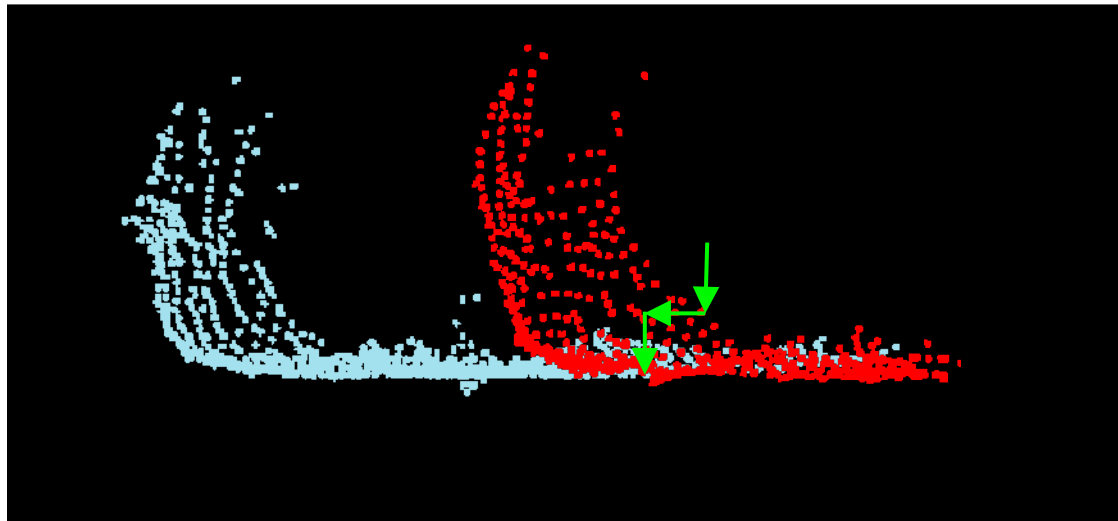


# ICP Baseline

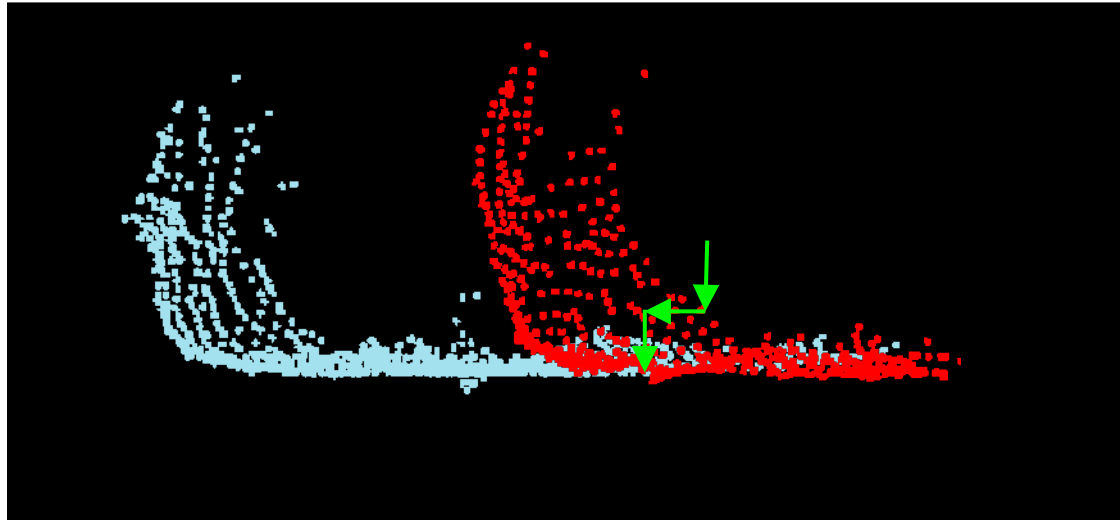




# ICP Baseline

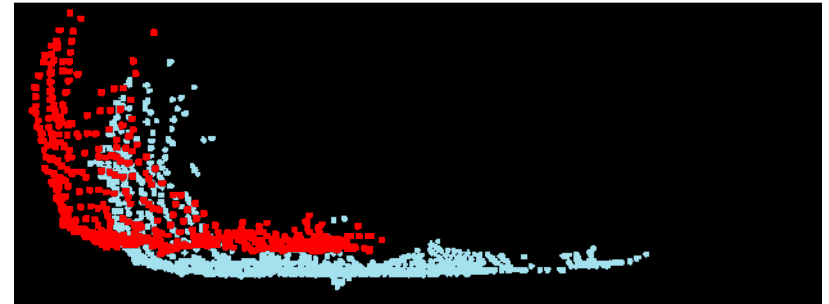
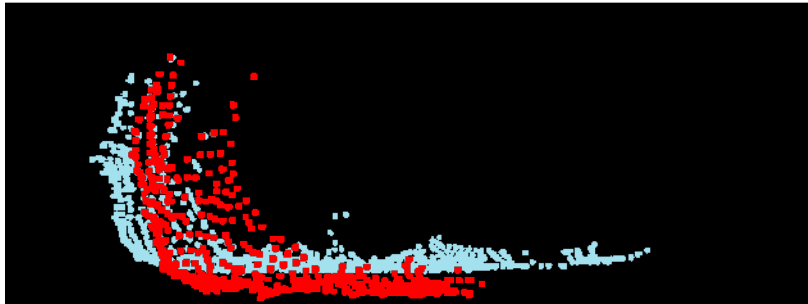
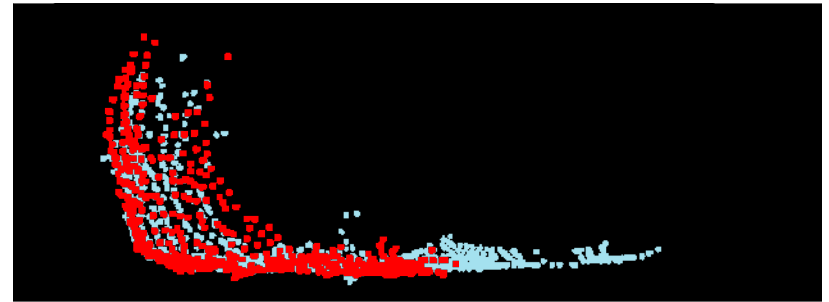
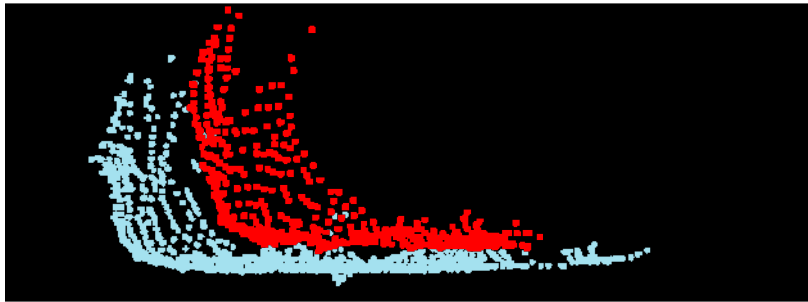


# ICP Baseline



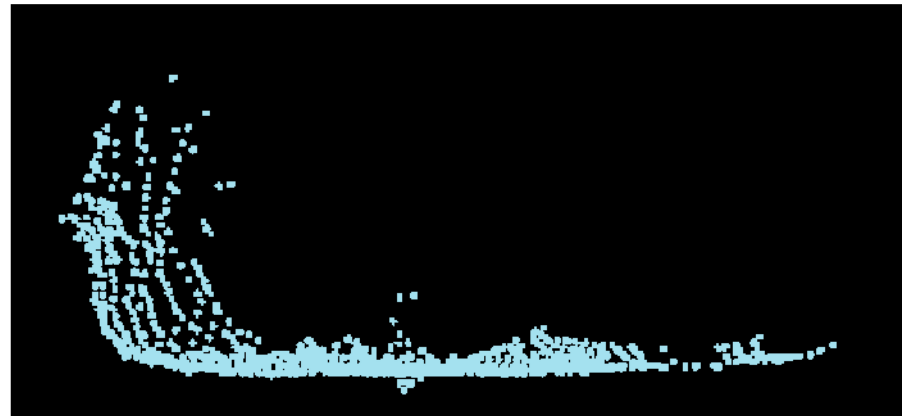
Local Search  $\rightarrow$  Poor Local Optimum!

# Tracking Probability



# Velocity Estimation

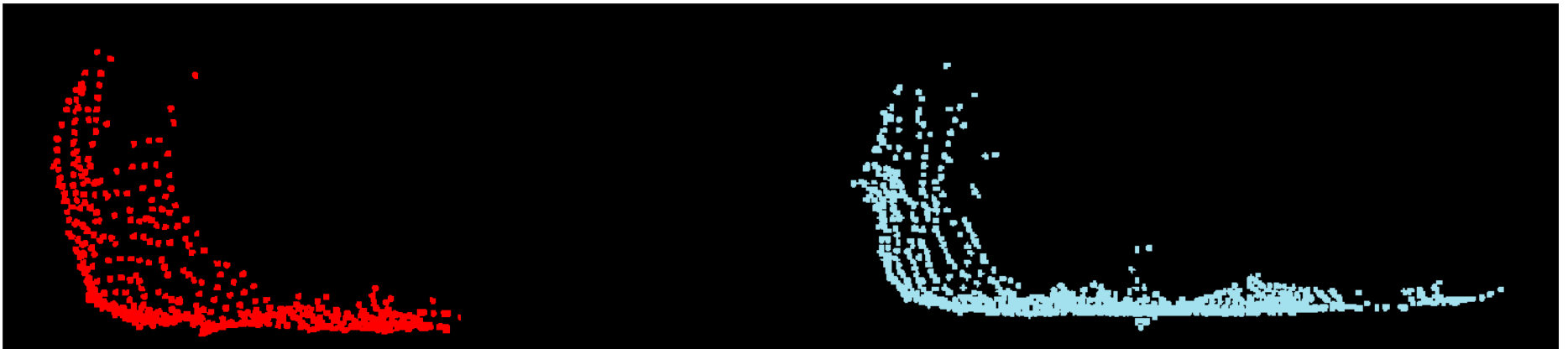
**t**



# Velocity Estimation

$t+1$

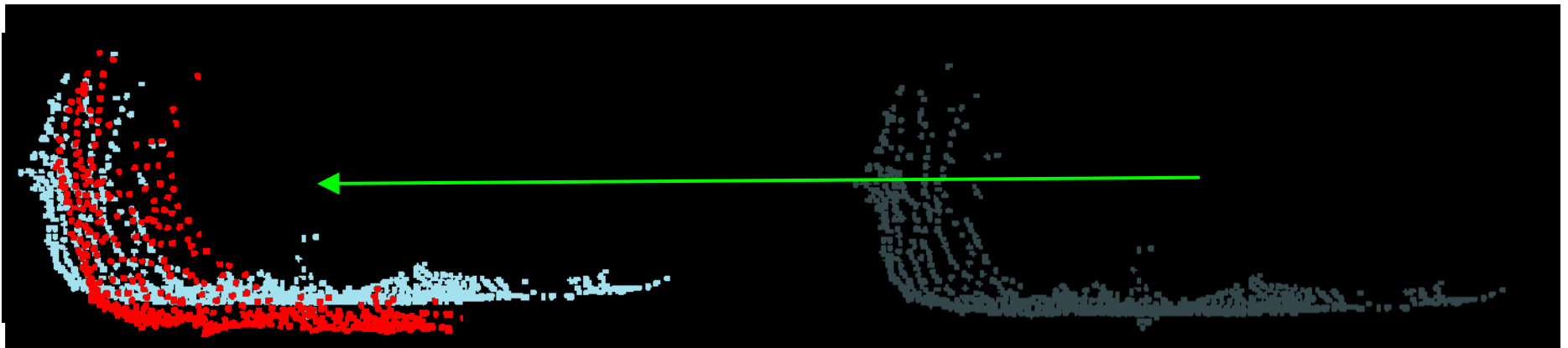
$t$



# Velocity Estimation

$t+1$

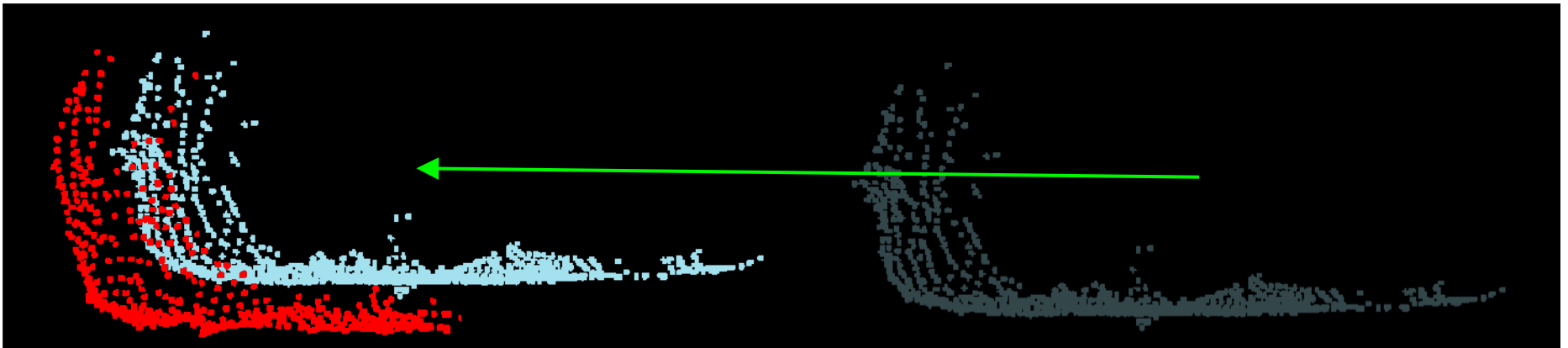
$t$



# Velocity Estimation

$t+1$

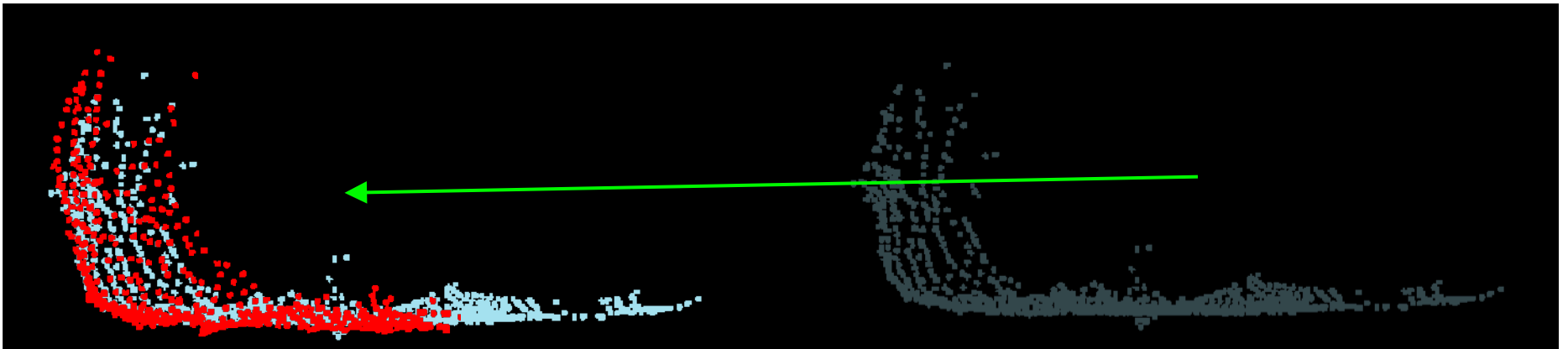
$t$



# Velocity Estimation

$t+1$

$t$

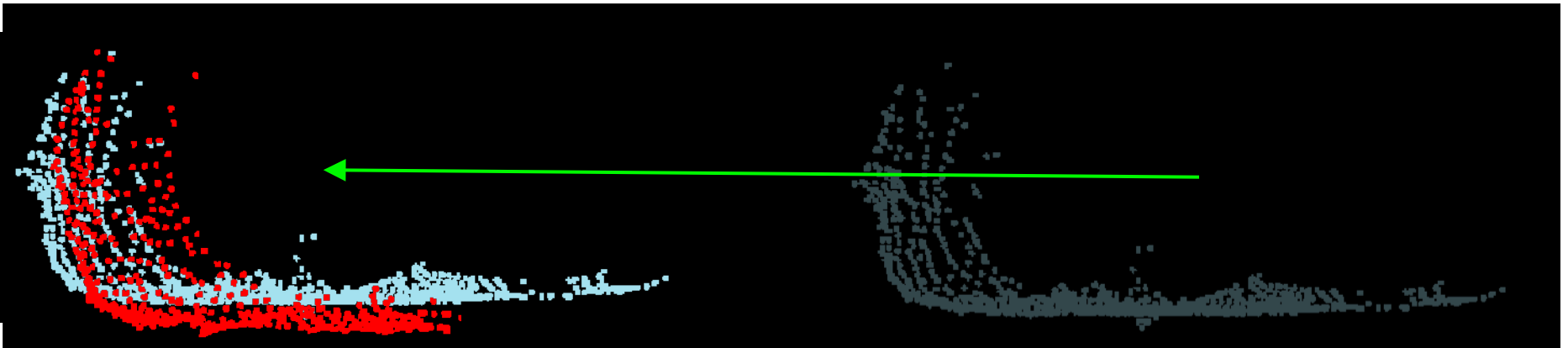




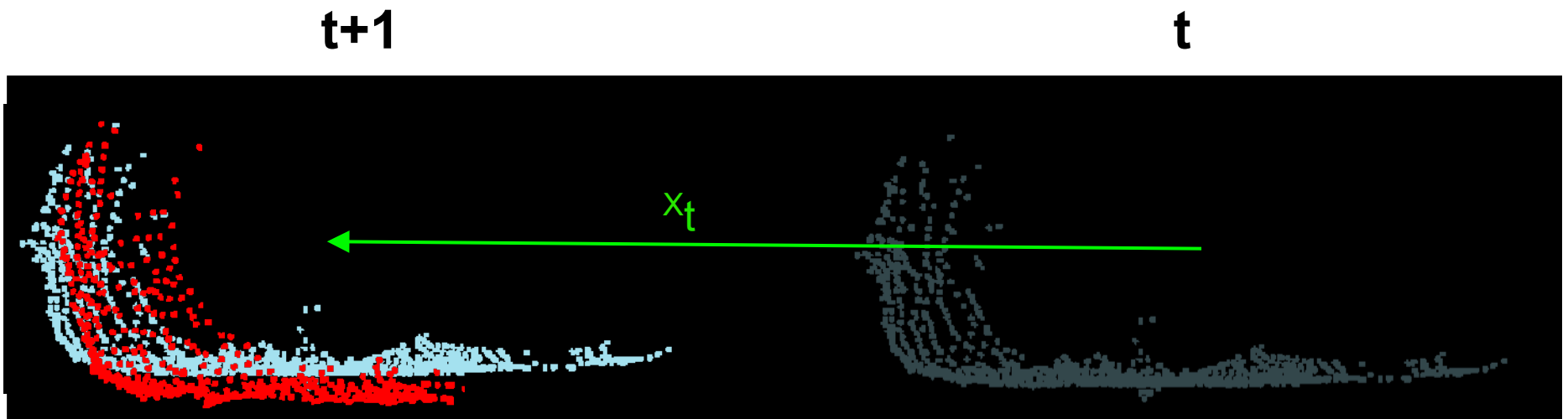
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$t+1$

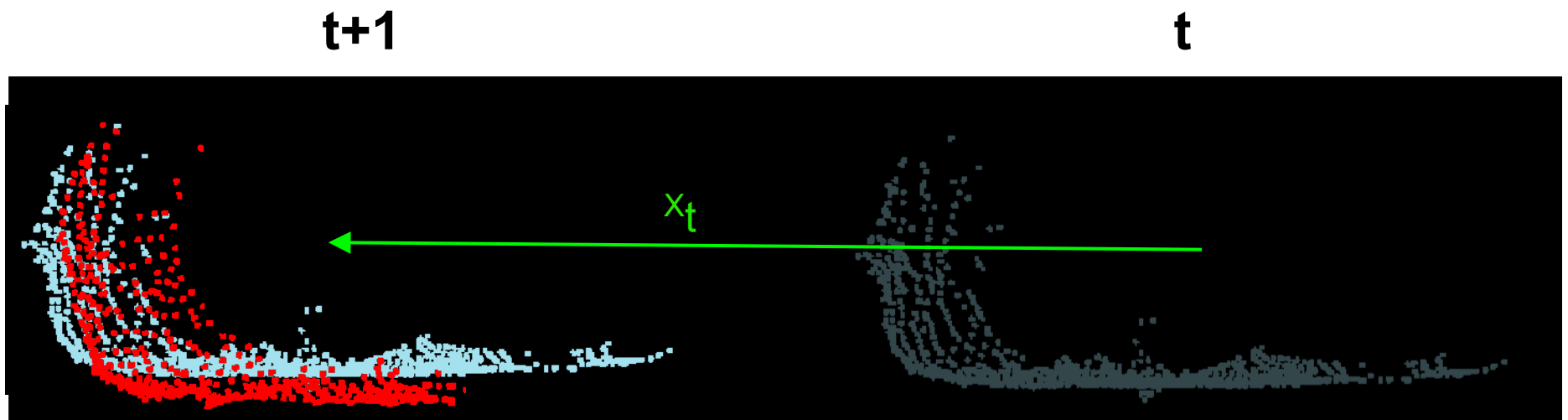
$t$



# Velocity Estimation



# Velocity Estimation



$$p(x_t \mid z_1 \dots z_t)$$

# Tracking Probability

$$p(x_t \mid z_1 \dots z_t) = \eta \underbrace{p(z_t \mid x_t, z_{t-1})}_{\text{Measurement Model}} \underbrace{p(x_t \mid z_1 \dots z_{t-1})}_{\text{Motion Model}}$$

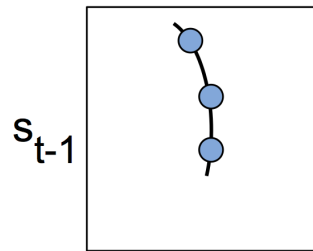
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$$p(x_t \mid z_1 \dots z_t) = \eta \underbrace{p(z_t \mid x_t, z_{t-1})}_{\text{Measurement Model}} \underbrace{p(x_t \mid z_1 \dots z_{t-1})}_{\text{Motion Model}}$$

Constant velocity  
Kalman filter

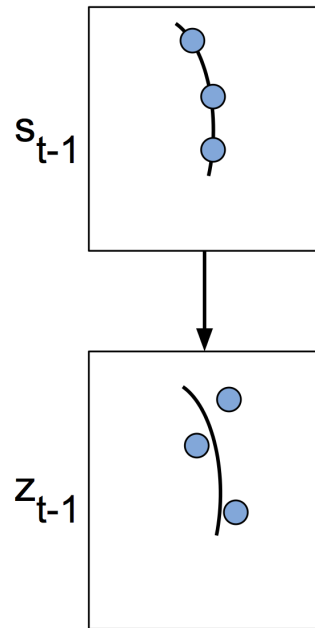
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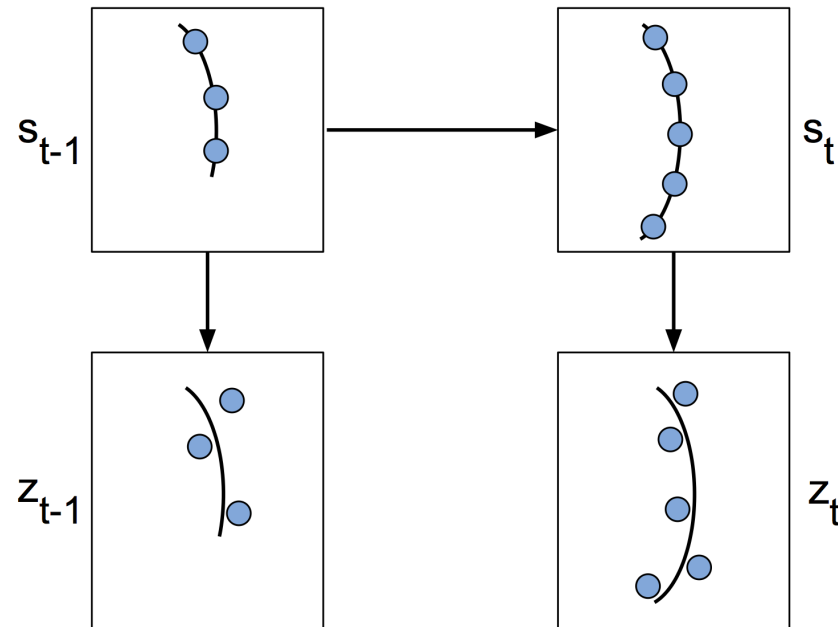
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# Tracking Probability

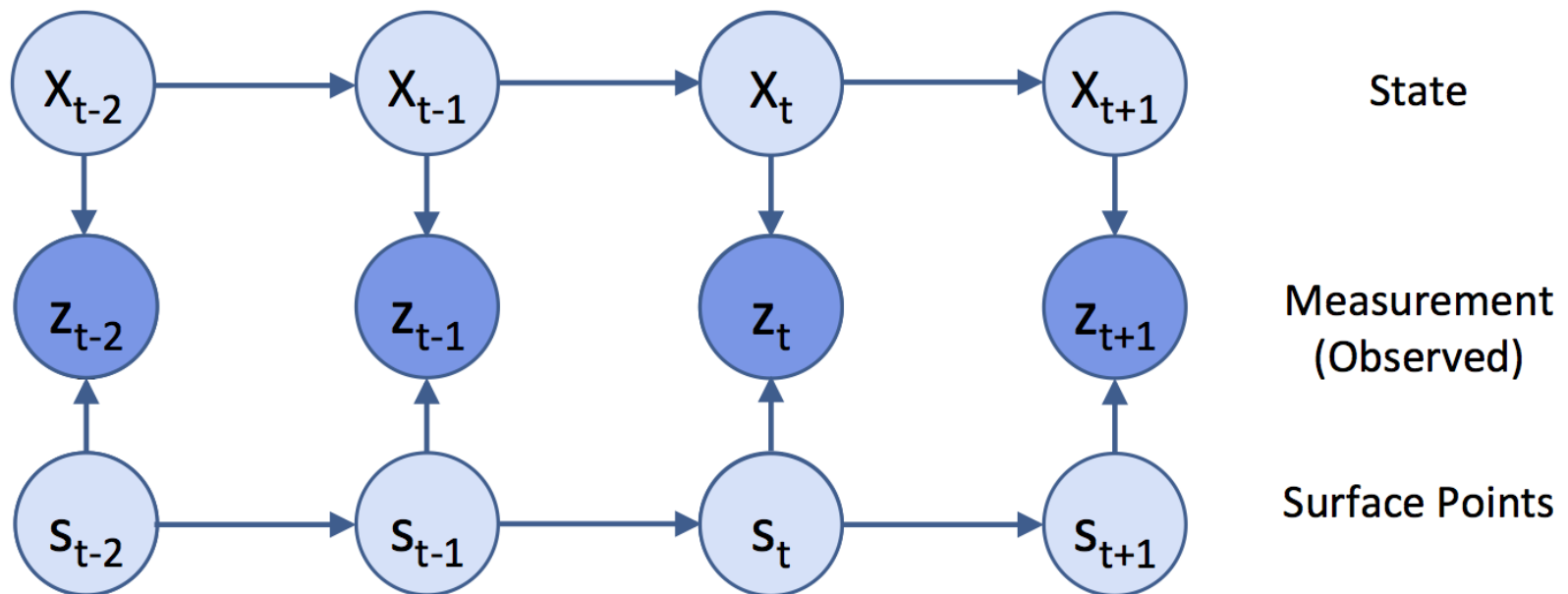
$$p(x_t \mid z_1 \dots z_t) = \underbrace{\eta p(z_t \mid x_t, z_{t-1})}_{\text{Measurement Model}} \underbrace{p(x_t \mid z_1 \dots z_{t-1})}_{\text{Motion Model}}$$





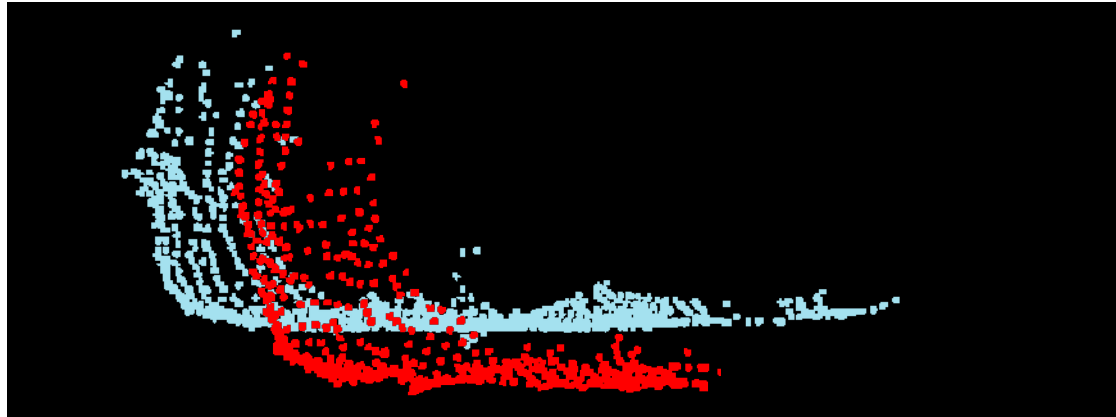
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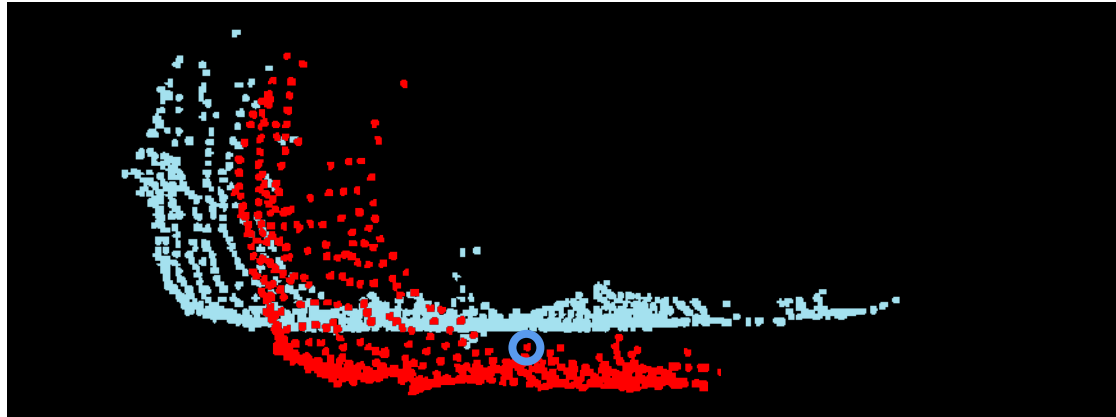
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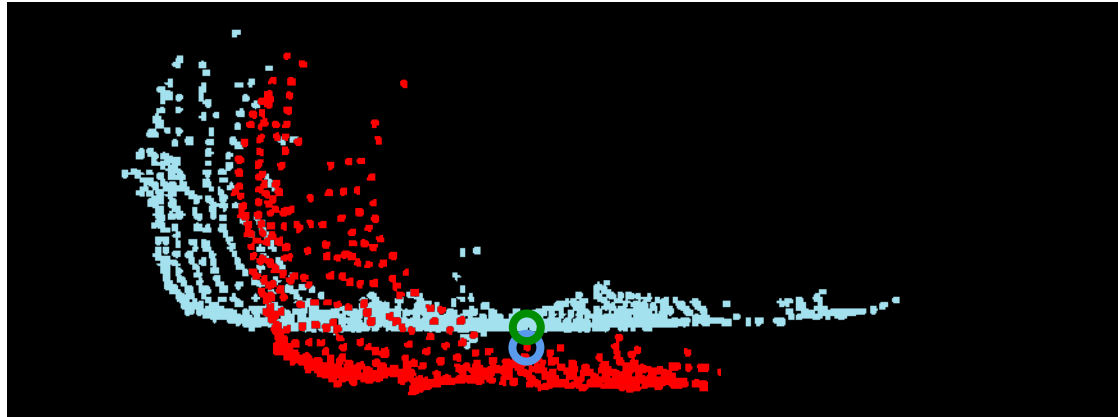
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$$p(x_t \mid z_1 \dots z_t) = \underbrace{\eta p(z_t \mid x_t, z_{t-1})}_{\substack{\text{Measurement} \\ \text{Model}}} p(x_t \mid z_1 \dots z_{t-1})_{\substack{\text{Motion Model}}}$$



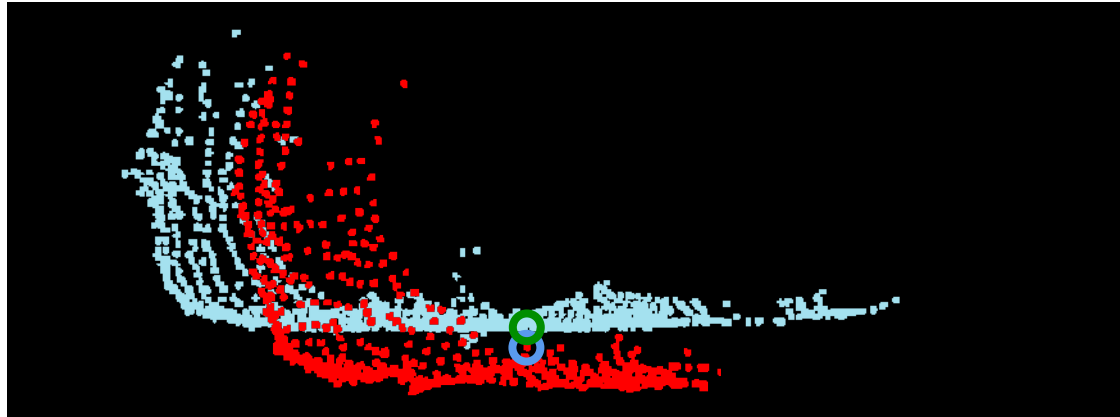
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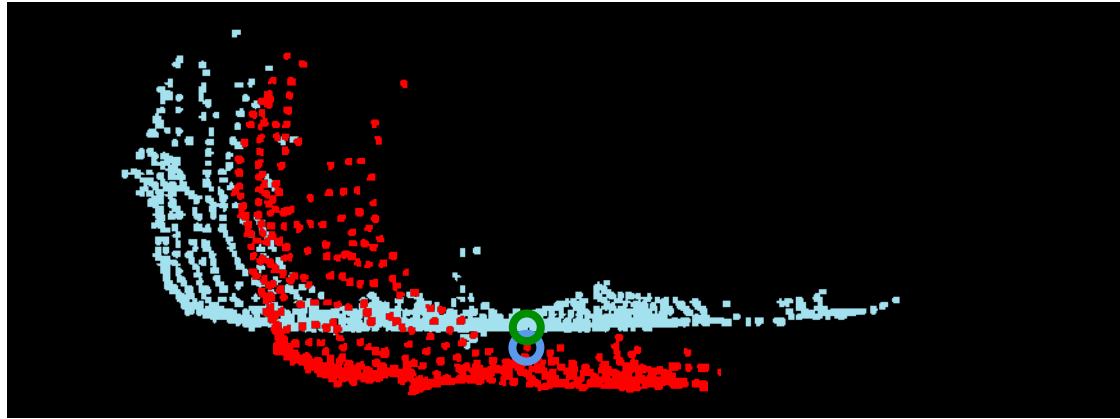
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$$= \eta \left( \prod_{z_i \in z_t} \exp \left( -\frac{1}{2} (\boxed{z_i} - \boxed{\bar{z}_j})^T \Sigma^{-1} (z_i - \bar{z}_j) \right) + k \right)$$

# Tracking Probability

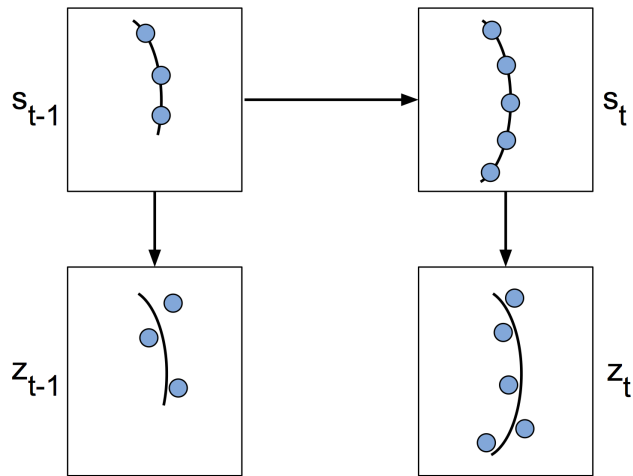
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# Tracking Probability

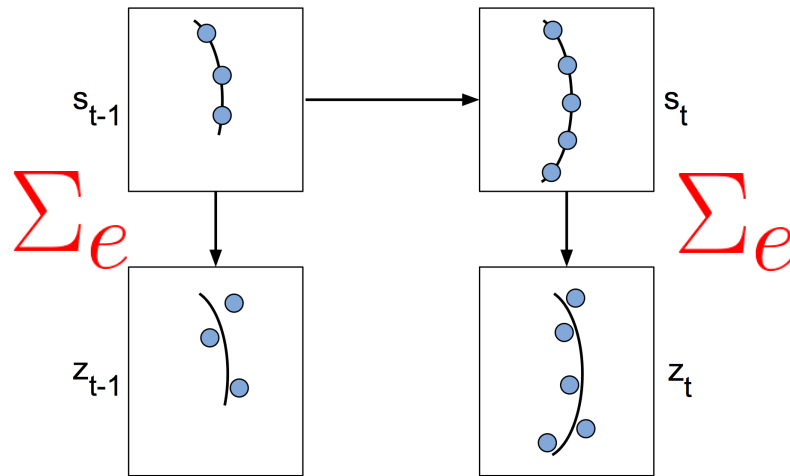
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$$= \eta \left( \prod_{z_i \in z_t} \exp \left( -\frac{1}{2} (\underbrace{z_i}_{\text{blue}} - \underbrace{\bar{z}_j}_{\text{green}})^T \Sigma^{-1} (z_i - \bar{z}_j) \right) + \underbrace{k}_{\text{purple}} \right)$$

# Tracking Probability

$$p(x_t \mid z_1 \dots z_t) = \underbrace{\eta p(z_t \mid x_t, z_{t-1})}_{\text{Measurement Model}} \underbrace{p(x_t \mid z_1 \dots z_{t-1})}_{\text{Motion Model}}$$

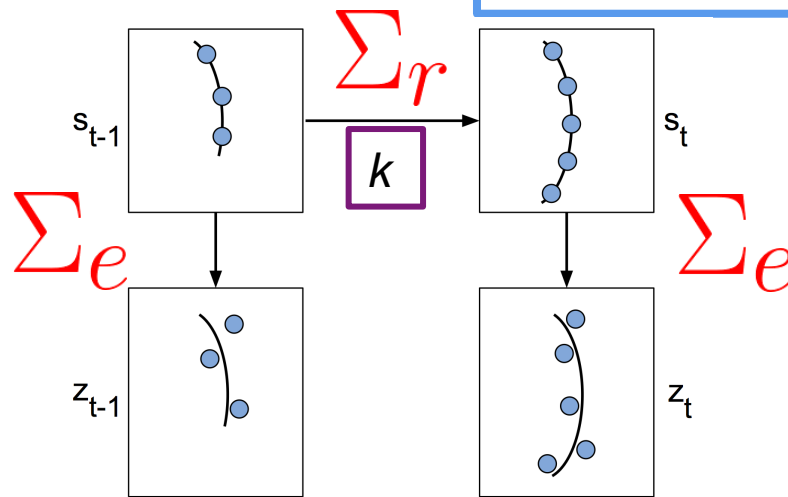


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# Tracking Probability

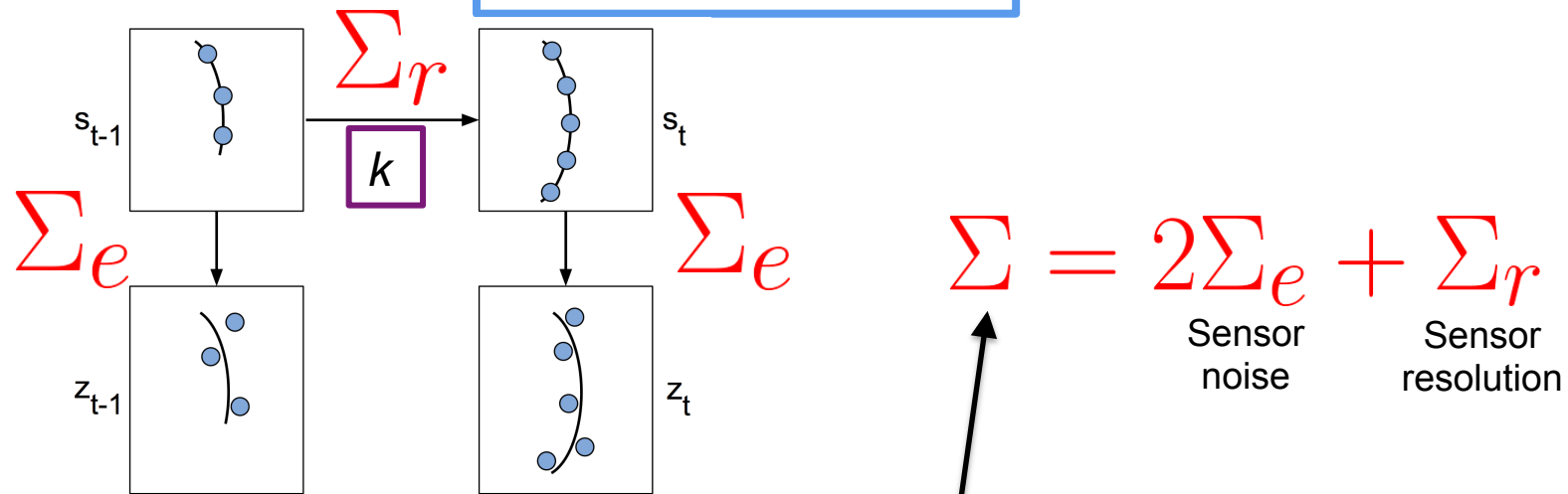
$$p(x_t \mid z_1 \dots z_t) = \underbrace{\eta p(z_t \mid x_t, z_{t-1})}_{\substack{\text{Measurement} \\ \text{Model}}} \underbrace{p(x_t \mid z_1 \dots z_{t-1})}_{\text{Motion Model}}$$



$$= \eta \left( \prod_{z_i \in z_t} \exp \left( -\frac{1}{2} (\underbrace{z_i}_{\text{blue box}} - \underbrace{\bar{z}_j}_{\text{green box}})^T \Sigma^{-1} (z_i - \bar{z}_j) \right) + \underbrace{k}_{\text{purple box}} \right)$$

# Tracking Probability

$$p(x_t \mid z_1 \dots z_t) = \underbrace{\eta p(z_t \mid x_t, z_{t-1})}_{\text{Measurement Model}} p(x_t \mid z_1 \dots z_{t-1})_{\text{Motion Model}}$$

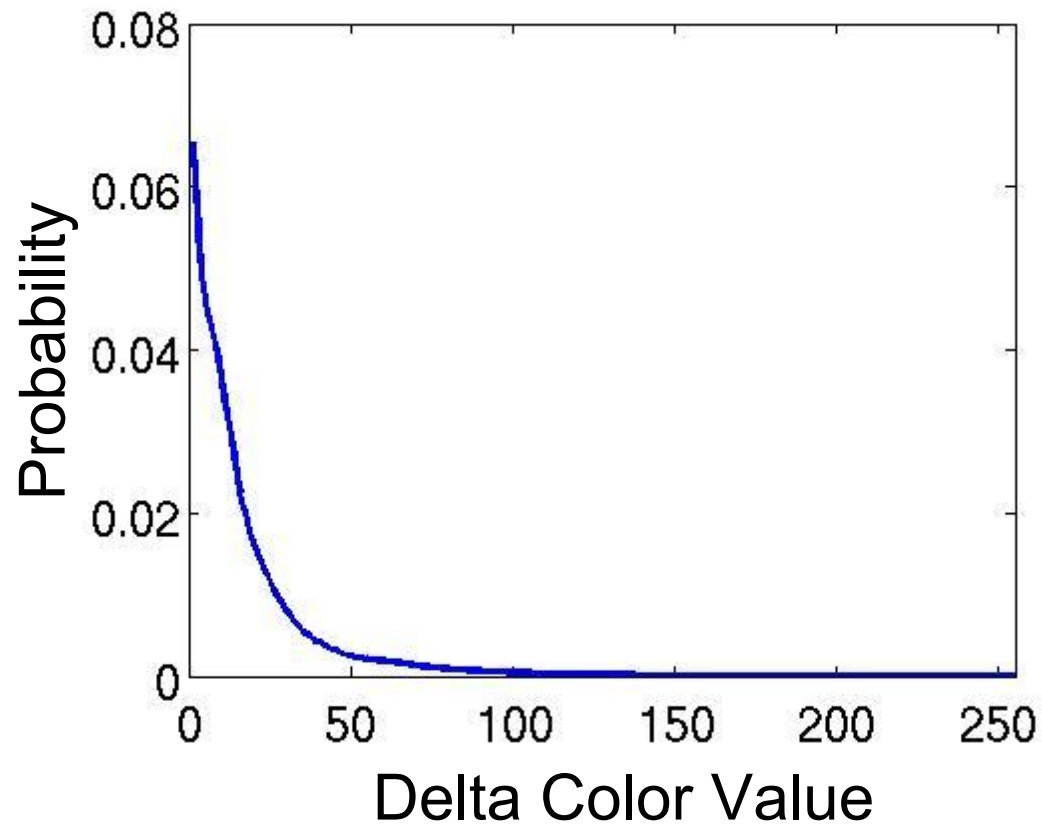


$$\Sigma = 2\underbrace{\Sigma_e}_{\text{Sensor noise}} + \underbrace{\Sigma_r}_{\text{Sensor resolution}}$$

$$= \eta \left( \prod_{z_i \in z_t} \exp \left( -\frac{1}{2} (\underbrace{z_i}_{\text{blue box}} - \underbrace{\bar{z}_j}_{\text{green box}})^T \Sigma^{-1} (z_i - \bar{z}_j) \right) + \underbrace{k}_{\text{purple box}} \right)$$



# Color Probability

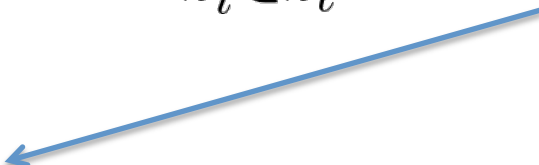


## Including Color

$$p(z_t \mid x_t, z_{t-1}) = \eta \prod_{z_i \in z_t} p_s(z_i \mid \bar{z}_j)$$

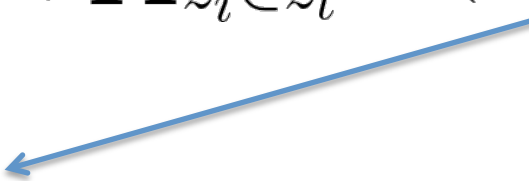
## Including Color

$$p(z_t \mid x_t, z_{t-1}) = \eta \prod_{z_i \in z_t} p_s(z_i \mid \bar{z}_j)$$


$$\eta \left( \prod_{z_i \in z_t} \exp\left(-\frac{1}{2}(z_i - \bar{z}_j)^T \Sigma^{-1}(z_i - \bar{z}_j)\right) + k \right)$$

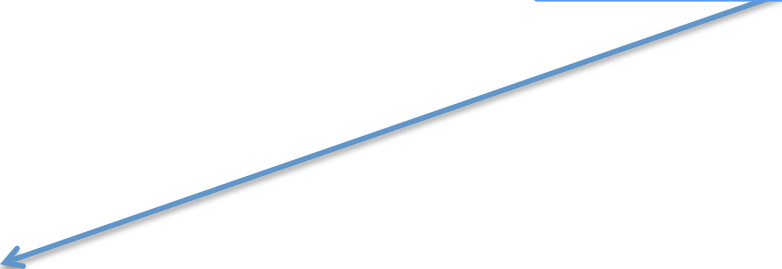
## Including Color

$$p(z_t \mid x_t, z_{t-1}) = \eta \prod_{z_i \in z_t} p_s(z_i \mid \bar{z}_j) p_c(z_i \mid \bar{z}_j)$$

$$\eta \left( \prod_{z_i \in z_t} \exp\left(-\frac{1}{2}(z_i - \bar{z}_j)^T \Sigma^{-1}(z_i - \bar{z}_j)\right) + k \right)$$


## Including Color

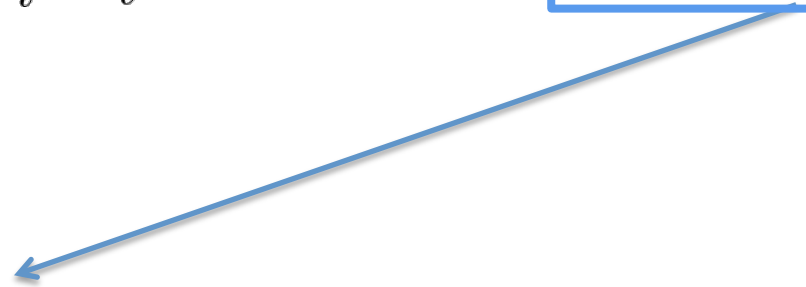
$$p(z_t \mid x_t, z_{t-1}) = \eta \prod_{z_i \in z_t} p_s(z_i \mid \bar{z}_j) p_c(z_i \mid \bar{z}_j)$$


$$p(C)p(z_t \mid \bar{z}_j, C) + P(\neg C)p(z_t \mid \bar{z}_j, \neg C)$$



## Including Color

$$p(z_t \mid x_t, z_{t-1}) = \eta \prod_{z_i \in z_t} p_s(z_i \mid \bar{z}_j) p_c(z_i \mid \bar{z}_j)$$



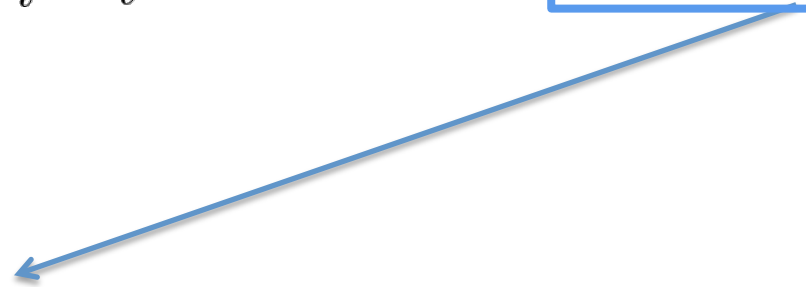
$$p(C)p(z_t \mid \bar{z}_j, C) + P(\neg C)p(z_t \mid \bar{z}_j, \neg C)$$



$$p_c \exp\left(\frac{-r^2}{2\sigma_c^2}\right)$$

## Including Color

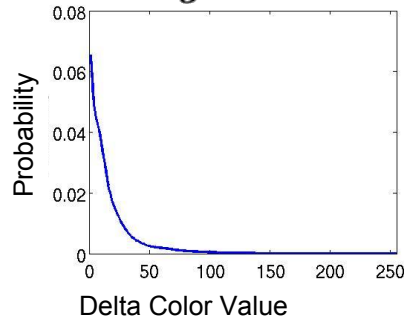
$$p(z_t \mid x_t, z_{t-1}) = \eta \prod_{z_i \in z_t} p_s(z_i \mid \bar{z}_j) p_c(z_i \mid \bar{z}_j)$$



$$p(C)p(z_t \mid \bar{z}_j, C) + P(\neg C)p(z_t \mid \bar{z}_j, \neg C)$$

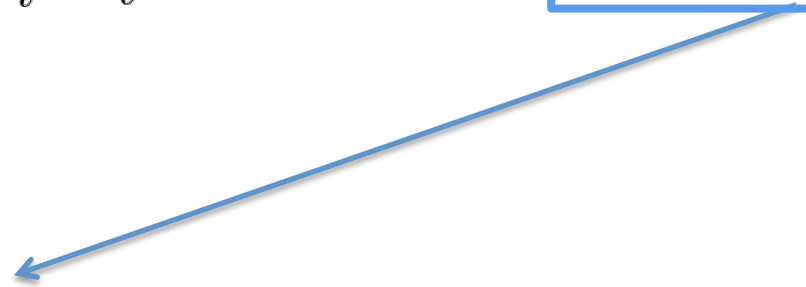
↓

$$p_c \exp\left(\frac{-r^2}{2\sigma_c^2}\right)$$



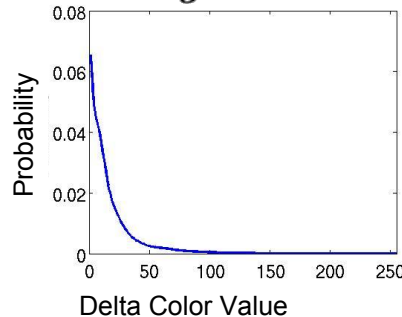
## Including Color

$$p(z_t \mid x_t, z_{t-1}) = \eta \prod_{z_i \in z_t} p_s(z_i \mid \bar{z}_j) p_c(z_i \mid \bar{z}_j)$$



$$p(C)p(z_t \mid \bar{z}_j, C) + P(\neg C)p(z_t \mid \bar{z}_j, \neg C)$$

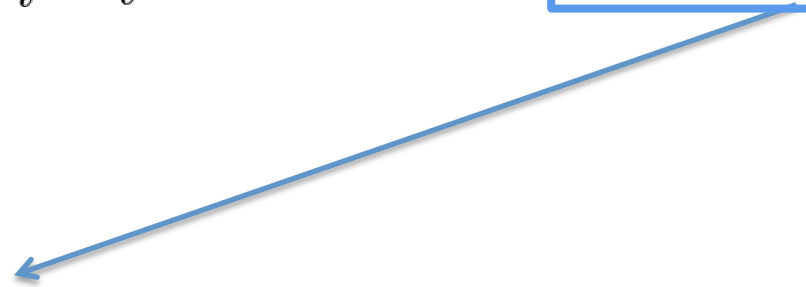
$$p_c \exp\left(\frac{-r^2}{2\sigma_c^2}\right)$$



$$1 - p_c \exp\left(\frac{-r^2}{2\sigma_c^2}\right)$$

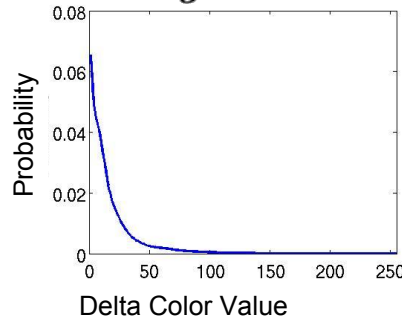
## Including Color

$$p(z_t \mid x_t, z_{t-1}) = \eta \prod_{z_i \in z_t} p_s(z_i \mid \bar{z}_j) p_c(z_i \mid \bar{z}_j)$$



$$p(C)p(z_t \mid \bar{z}_j, C) + P(\neg C)p(z_t \mid \bar{z}_j, \neg C)$$

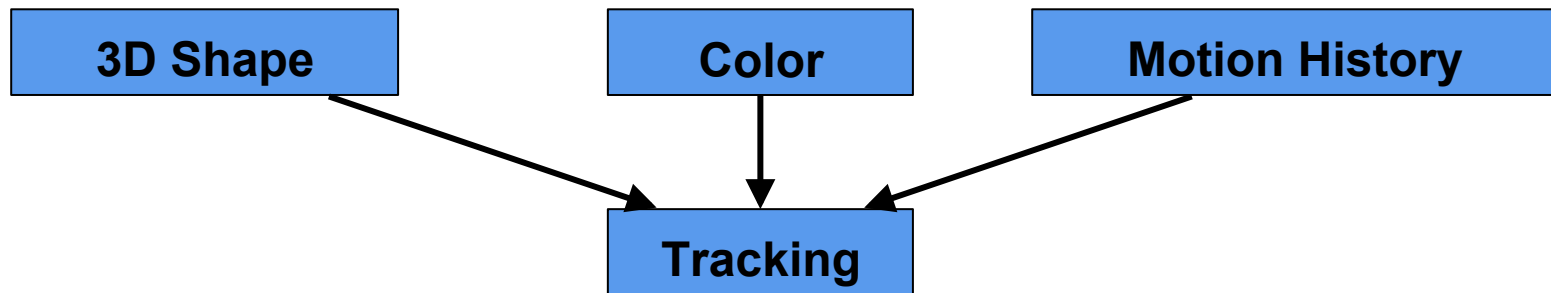
$$p_c \exp\left(\frac{-r^2}{2\sigma_c^2}\right)$$



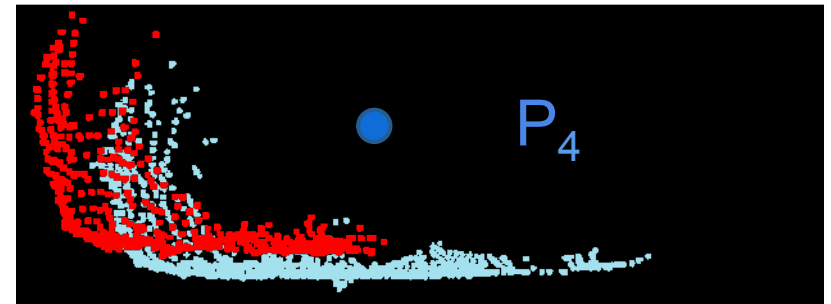
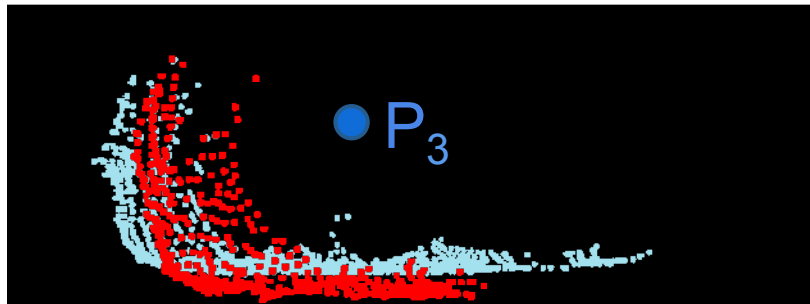
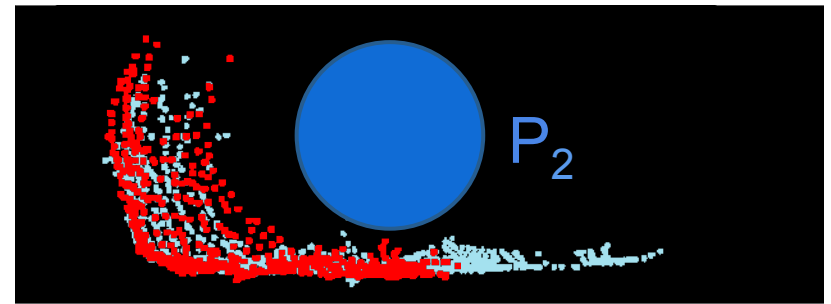
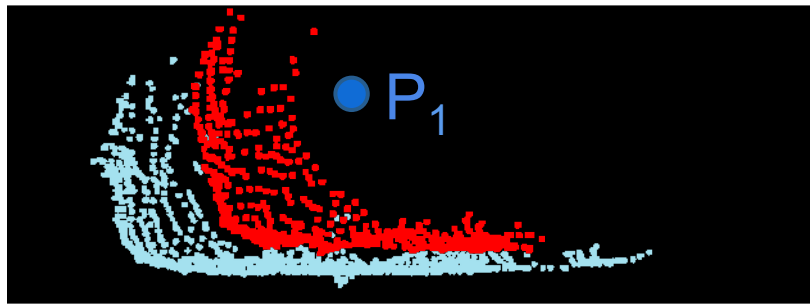
$$1 - p_c \exp\left(\frac{-r^2}{2\sigma_c^2}\right)$$

$$\frac{1}{255}$$

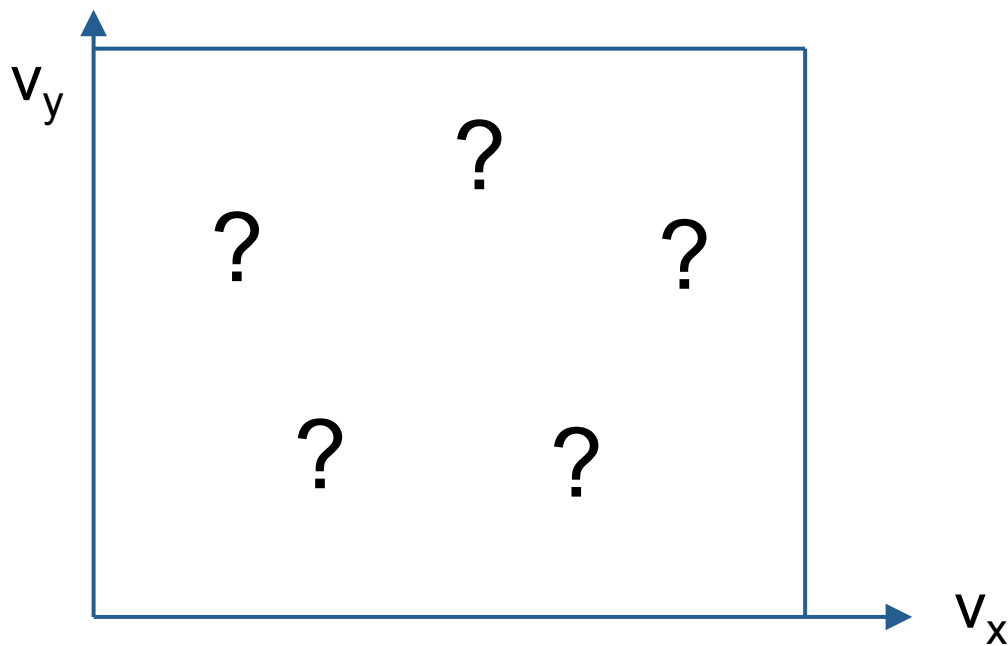
# Probabilistic Framework



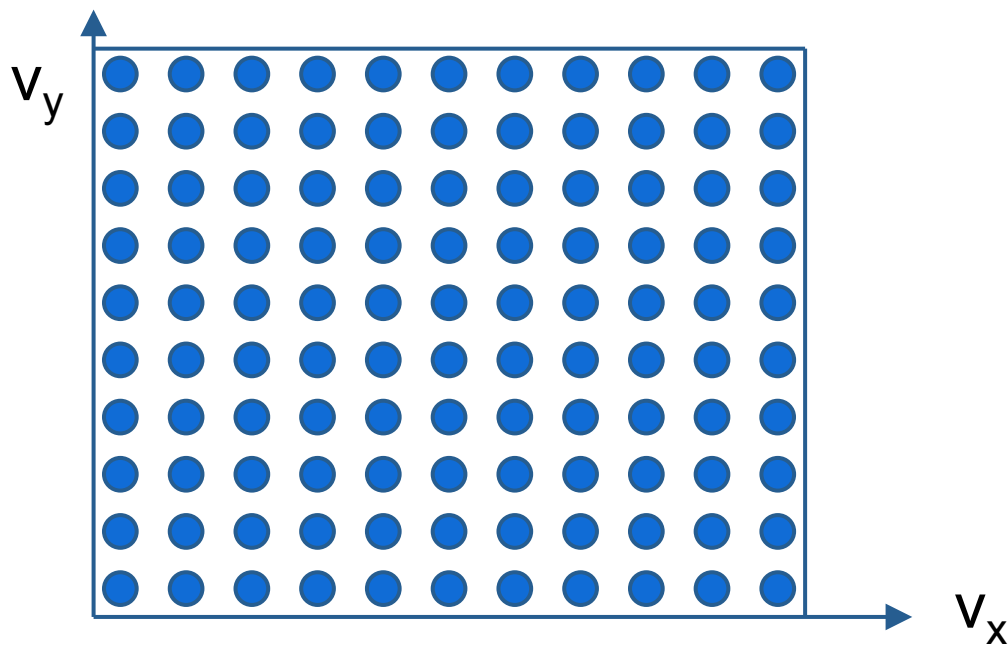
# Tracking Probability



# Tracking Probability

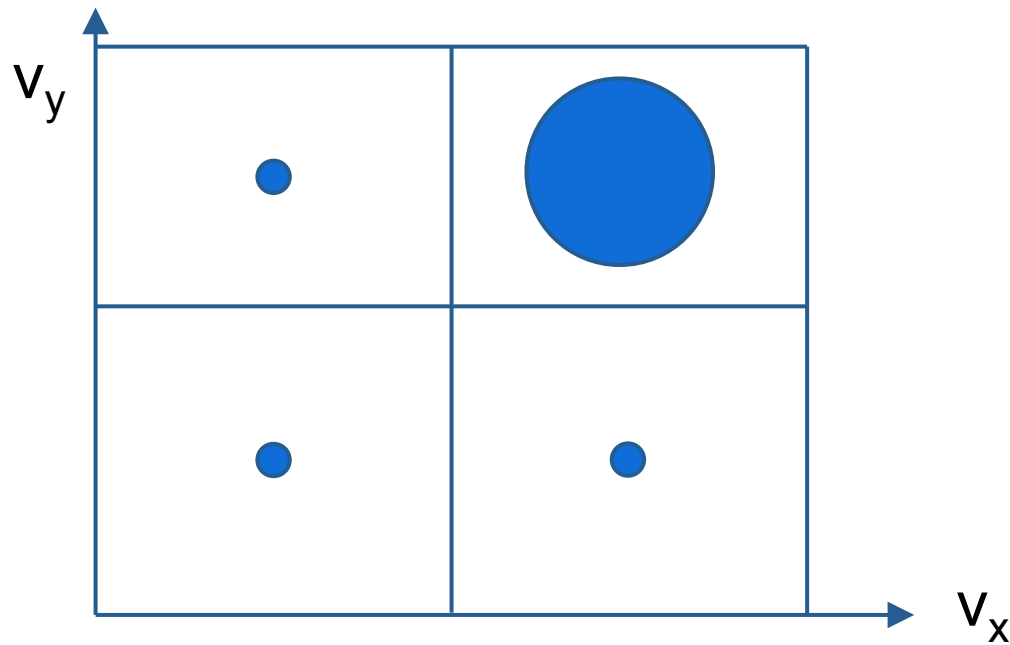


# Tracking Probability

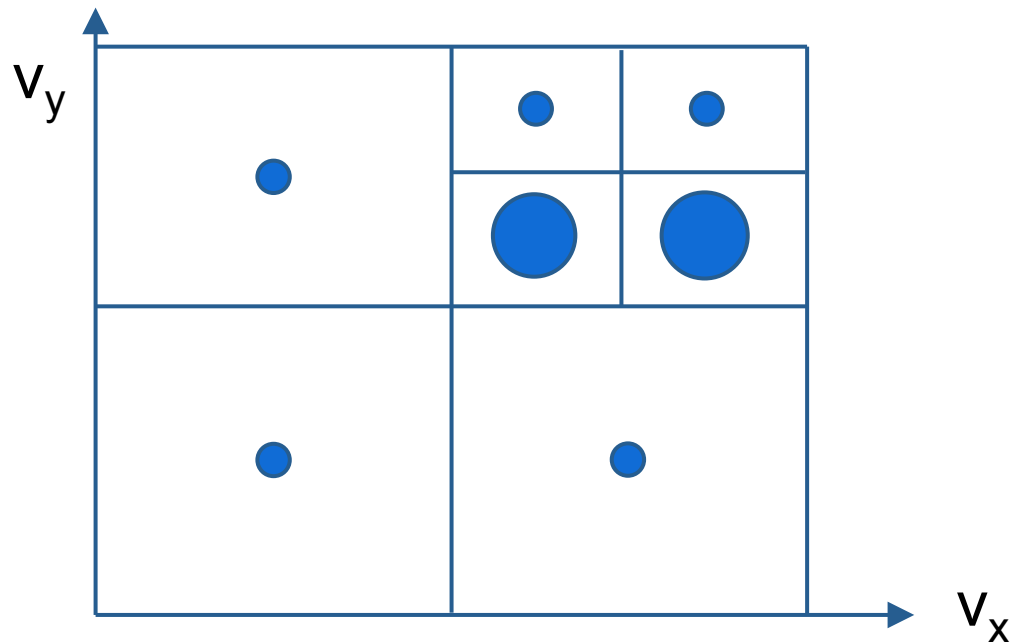




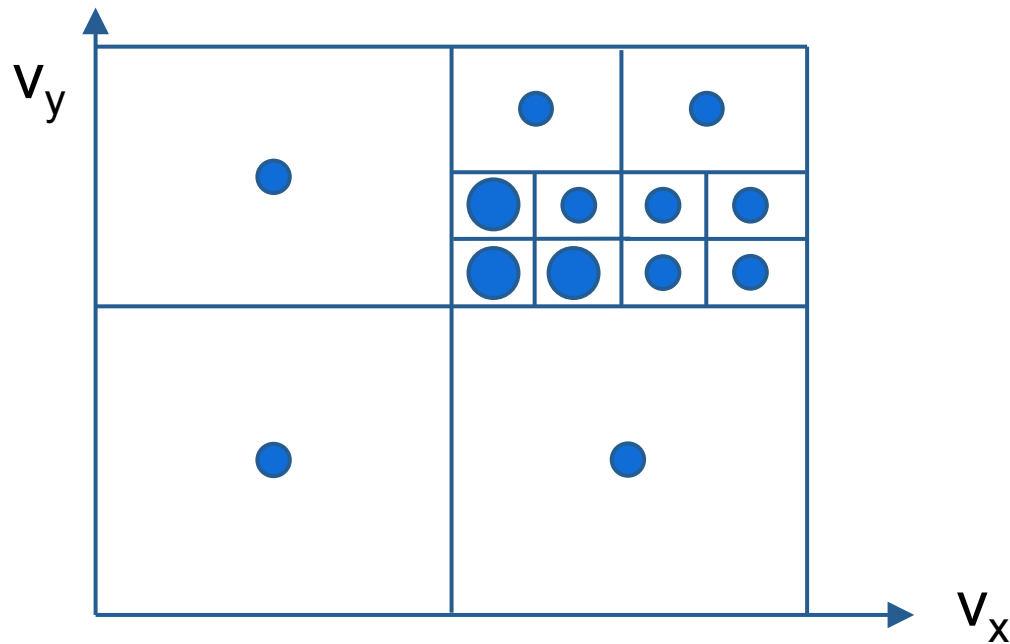
# Dynamic Decomposition



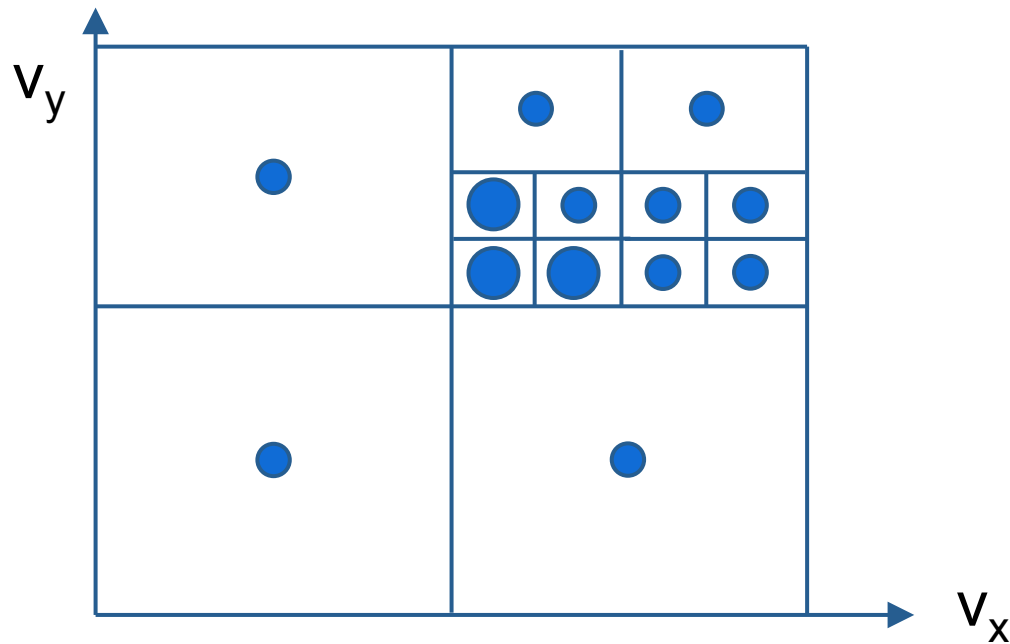
# Dynamic Decomposition



# Dynamic Decomposition

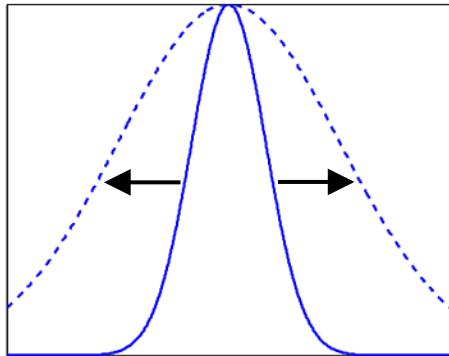
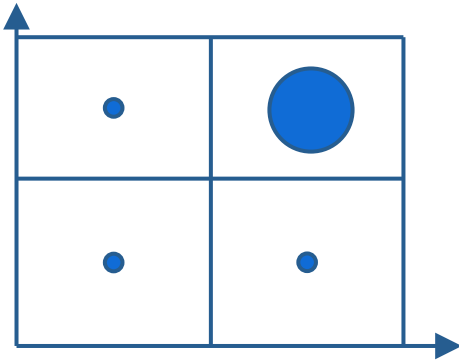


# Dynamic Decomposition



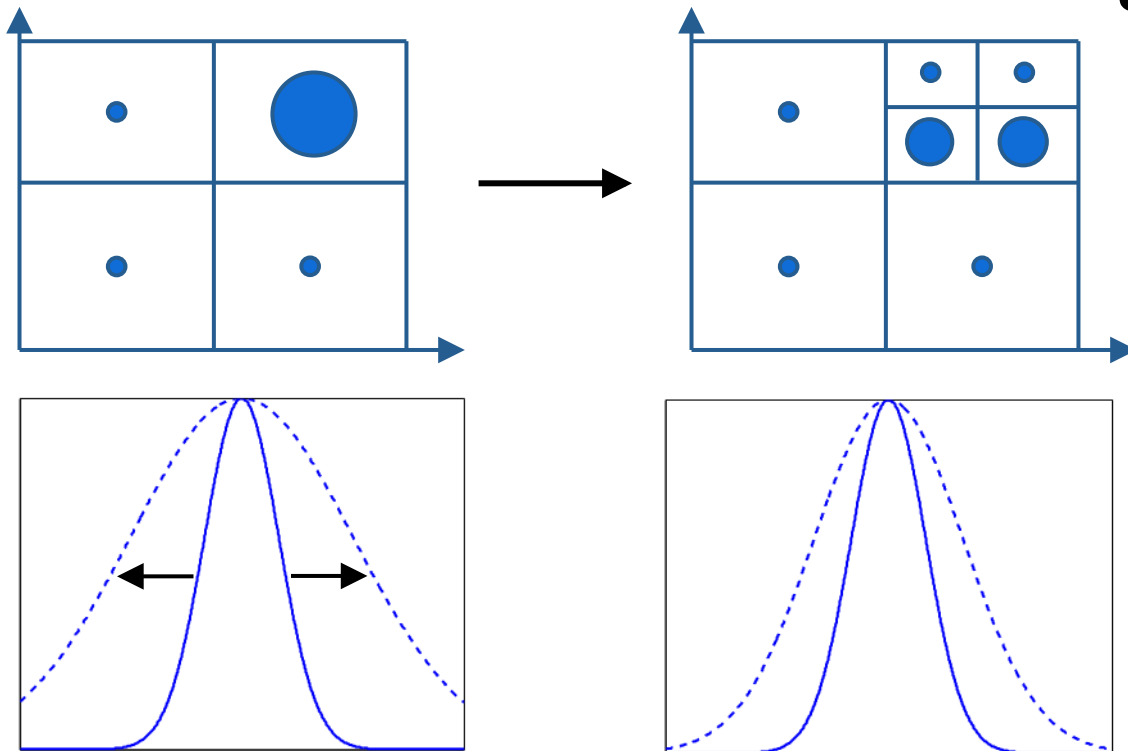
Derived from minimizing KL-divergence between approximate distribution and true posterior

# Annealing



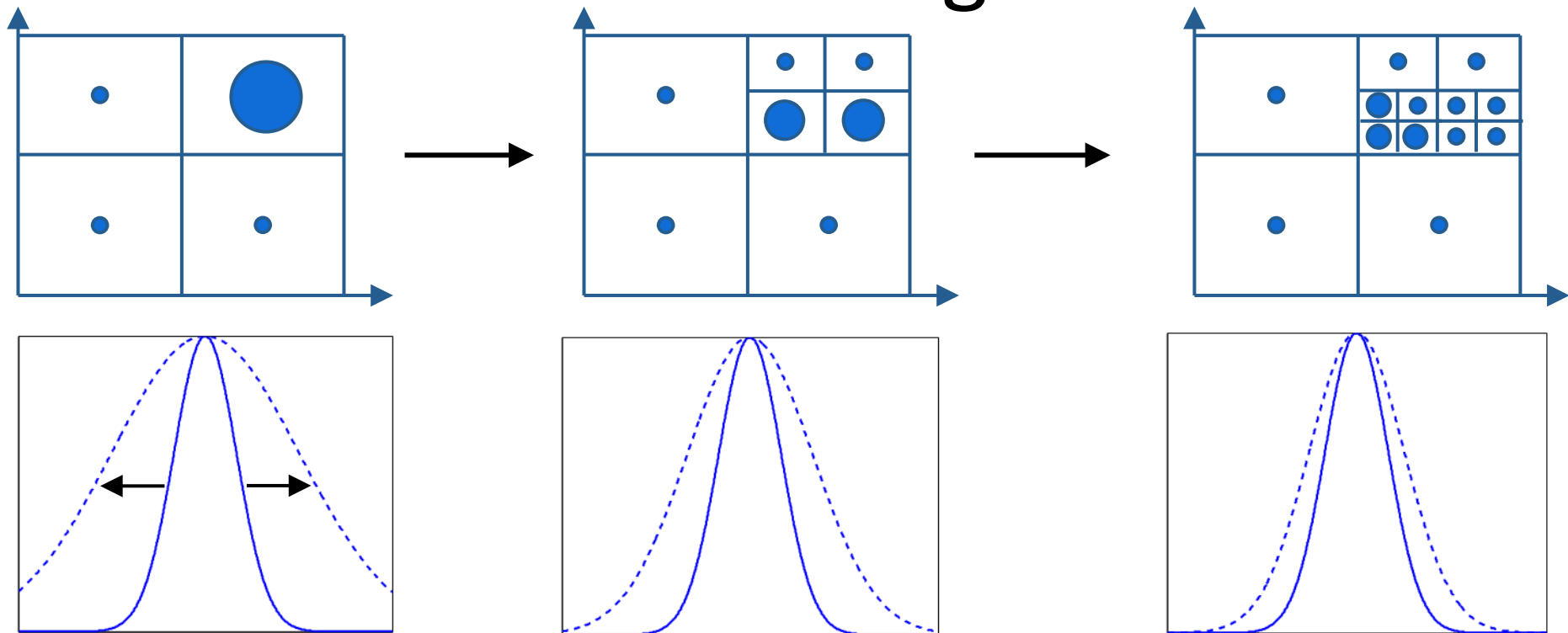
Inflate the  
measurement model

# Annealing



Inflate the  
measurement model

# Annealing



Inflate the  
measurement model

# Algorithm

1. For each hypothesis

A. Compute the probability of the alignment

$$p(x_t \mid z_1 \dots z_t) = \eta \underbrace{p(z_t \mid x_t, z_{t-1})}_{\text{Measurement Model}} \underbrace{p(x_t \mid z_1 \dots z_{t-1})}_{\text{Motion Model}}$$



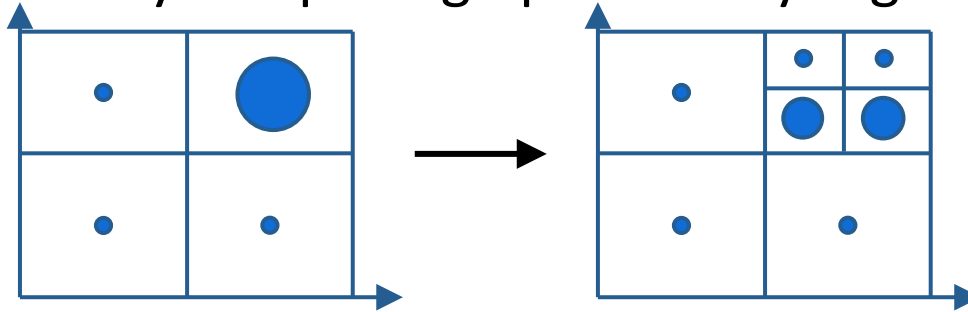
# Algorithm

1. For each hypothesis

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B. Finely sample high probability regions



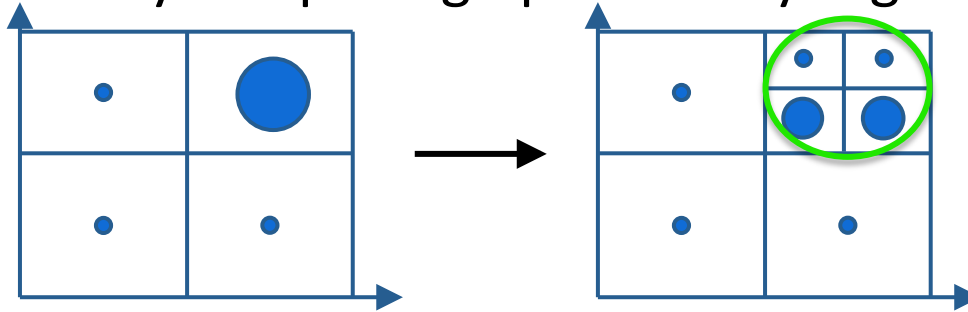
# Algorithm

1. For each hypothesis

A. Compute the probability of the alignment

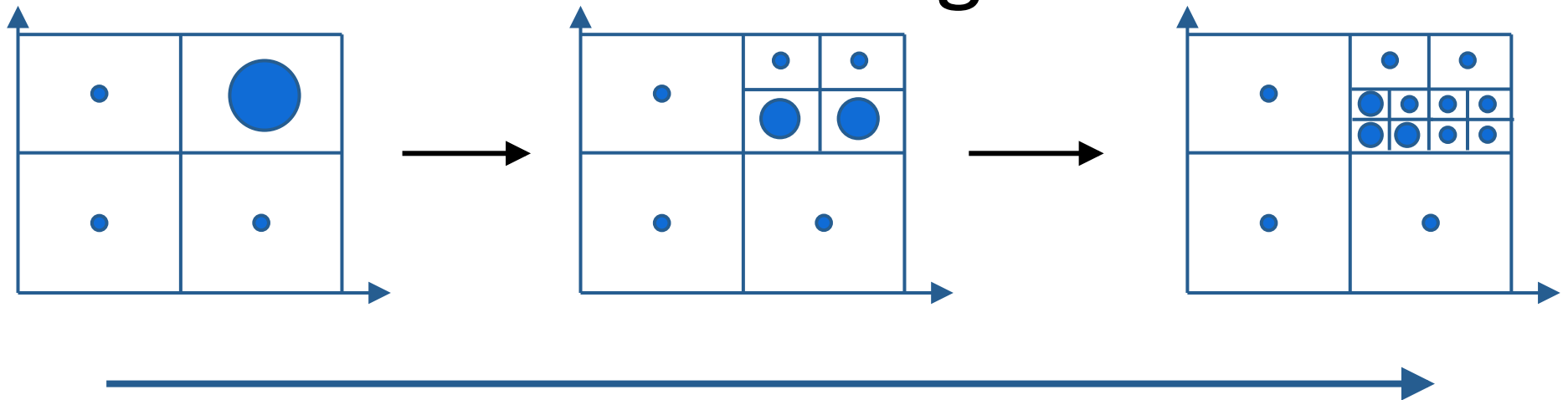
$$p(x_t \mid z_1 \dots z_t) = \eta \underbrace{p(z_t \mid x_t, z_{t-1})}_{\text{Measurement Model}} \underbrace{p(x_t \mid z_1 \dots z_{t-1})}_{\text{Motion Model}}$$

B. Finely sample high probability regions



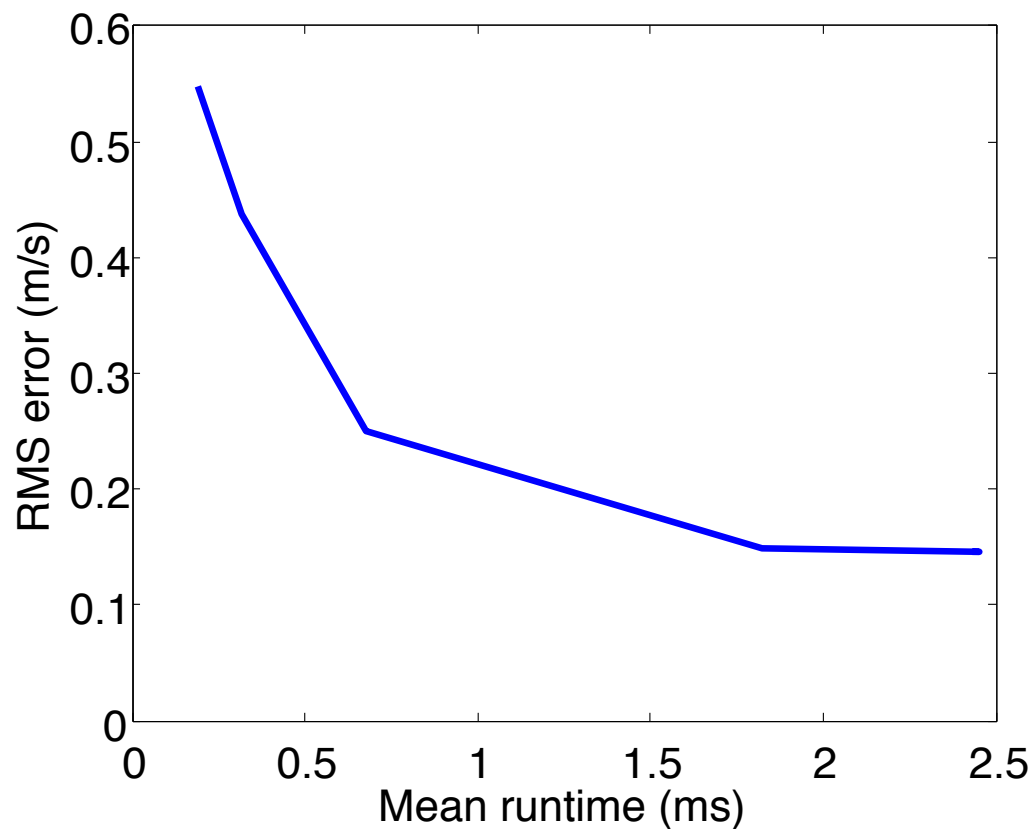
C. Go to step 1 to compute the probability of new hypotheses

# Annealing

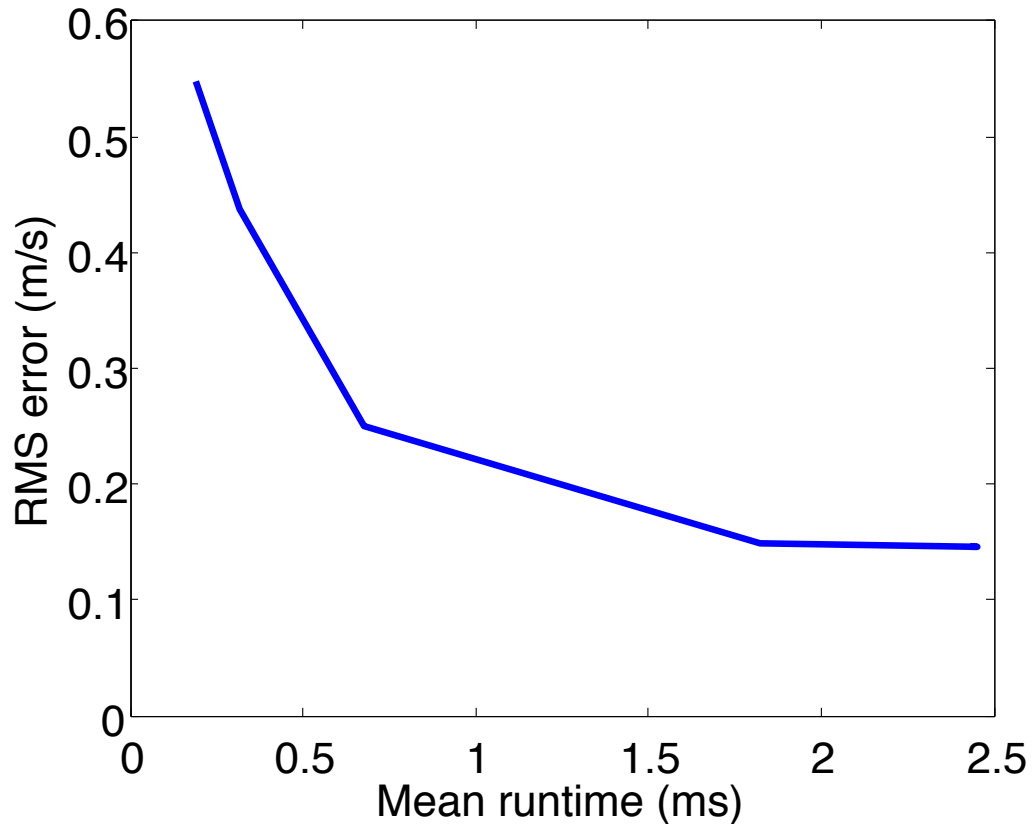


More time  
More accurate

# Anytime Tracker

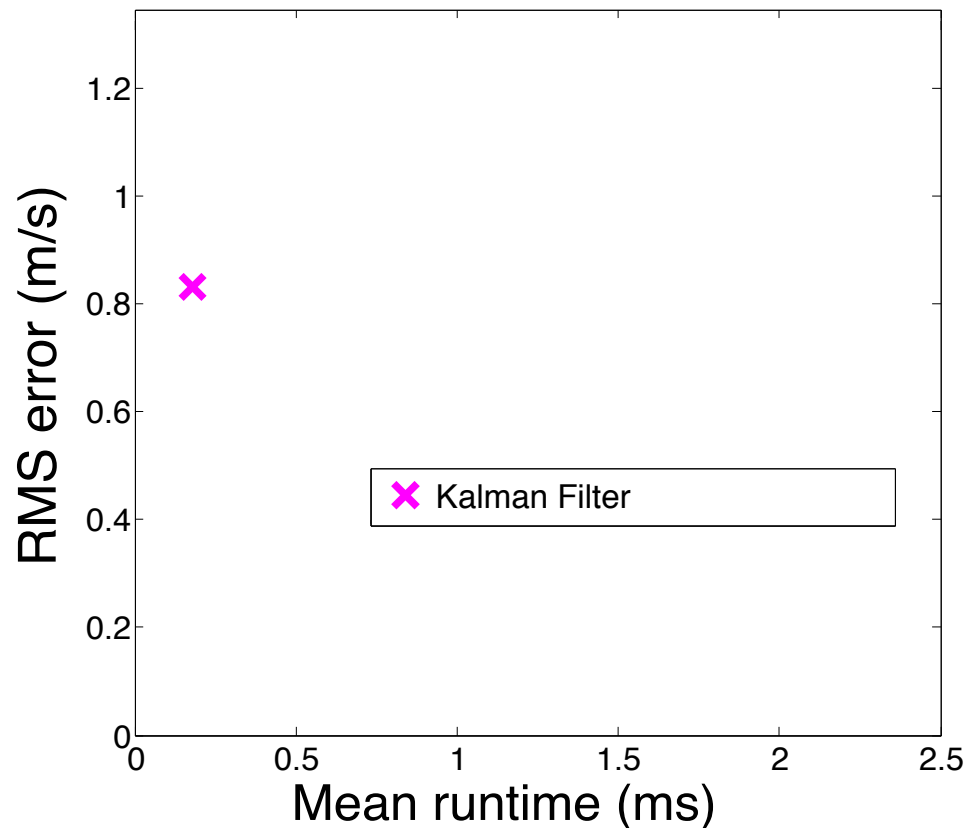


# Anytime Tracker

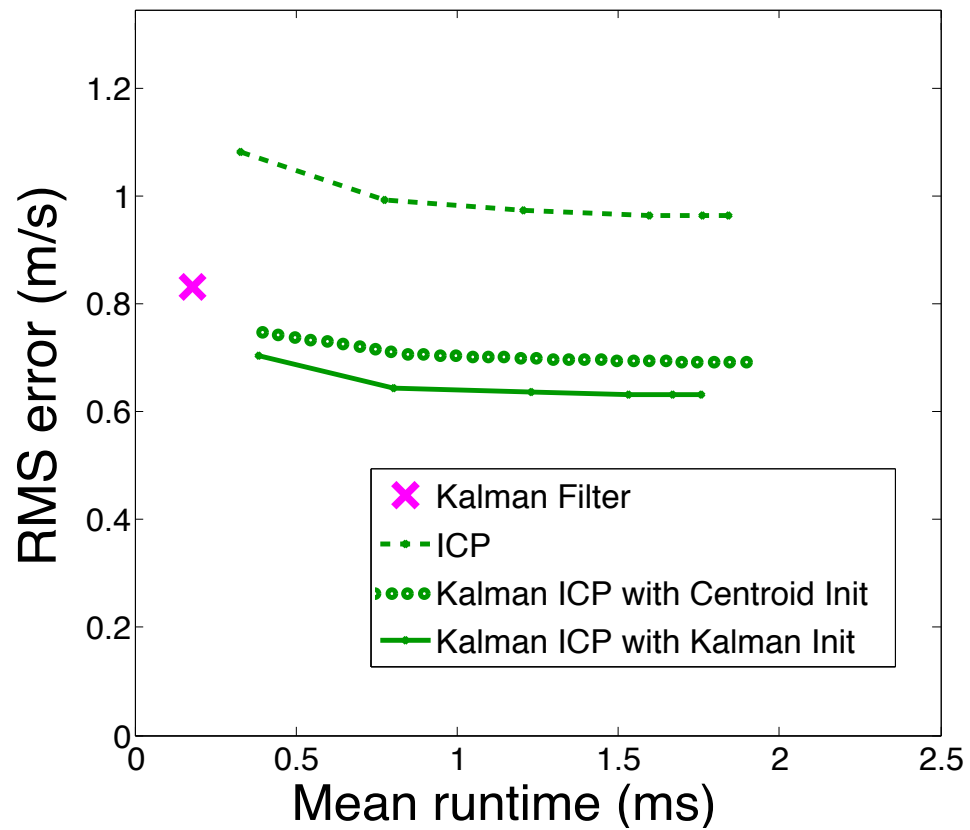


Choose runtime based on:  
Total runtime requirements  
Importance of tracked object  
...

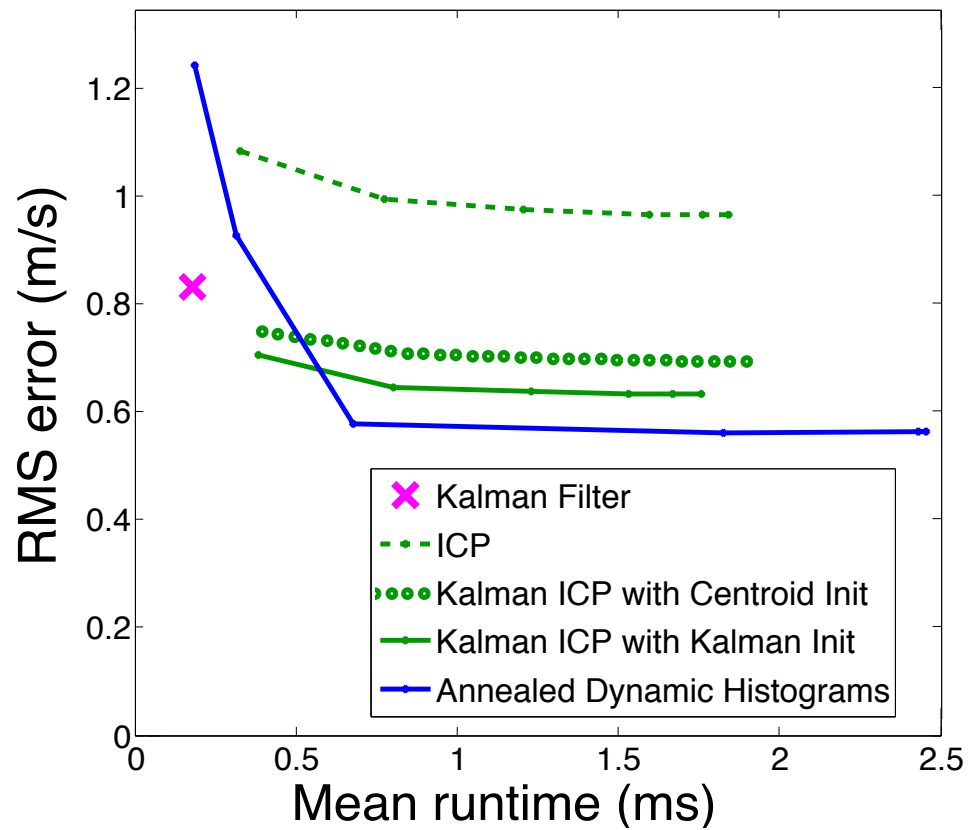
# Comparisons



# Comparisons

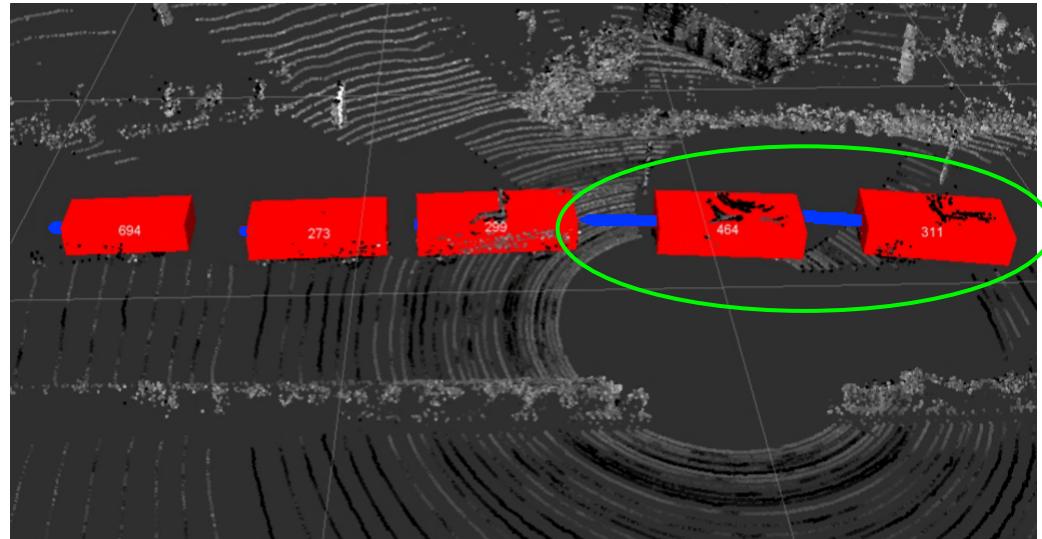


# Comparisons

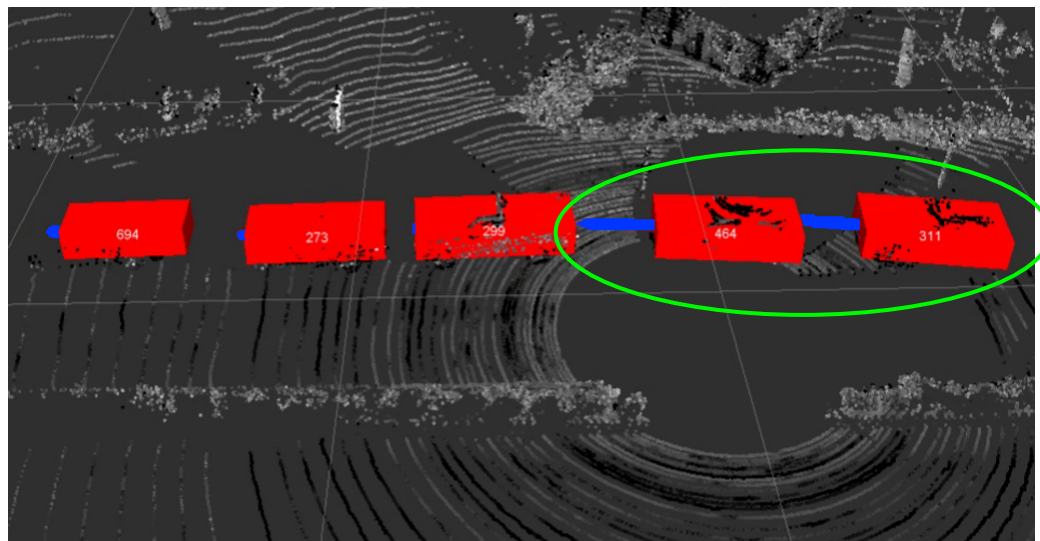




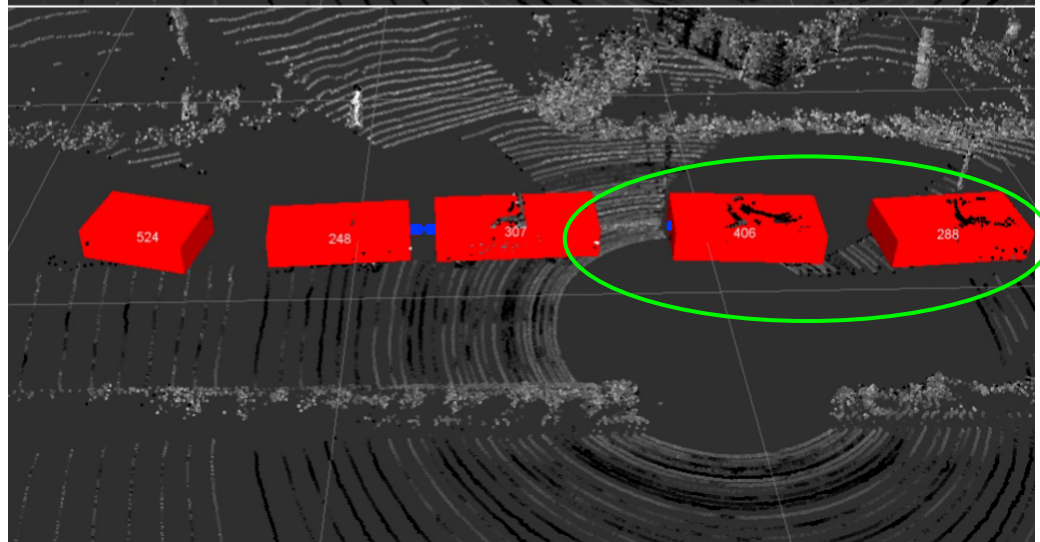
# Kalman Filter



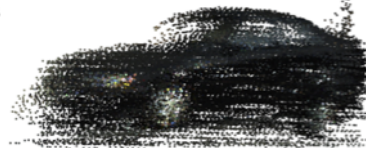
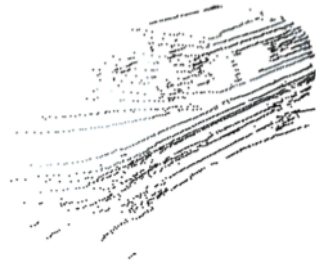
**Kalman  
Filter**



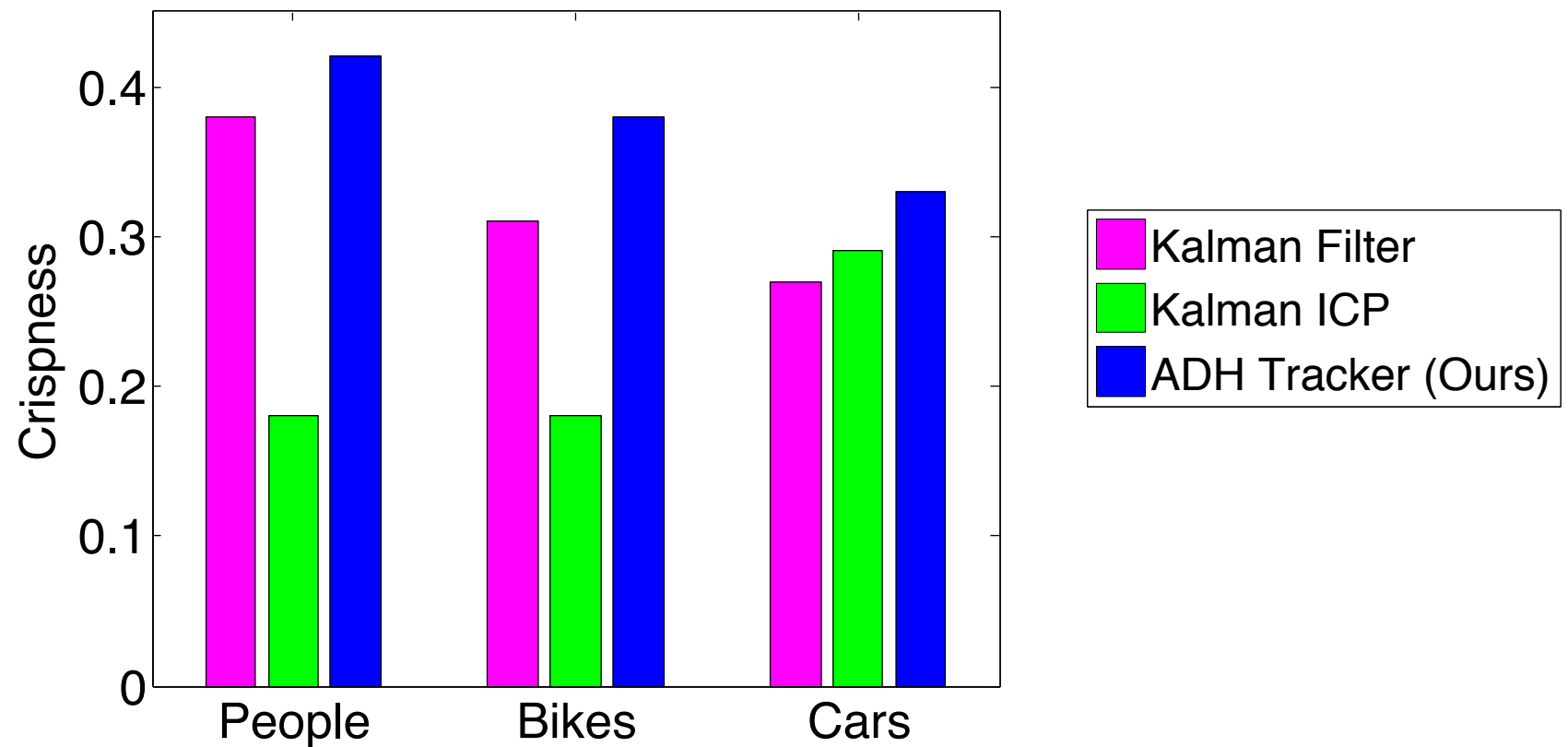
**ADH Tracker  
(Ours)**



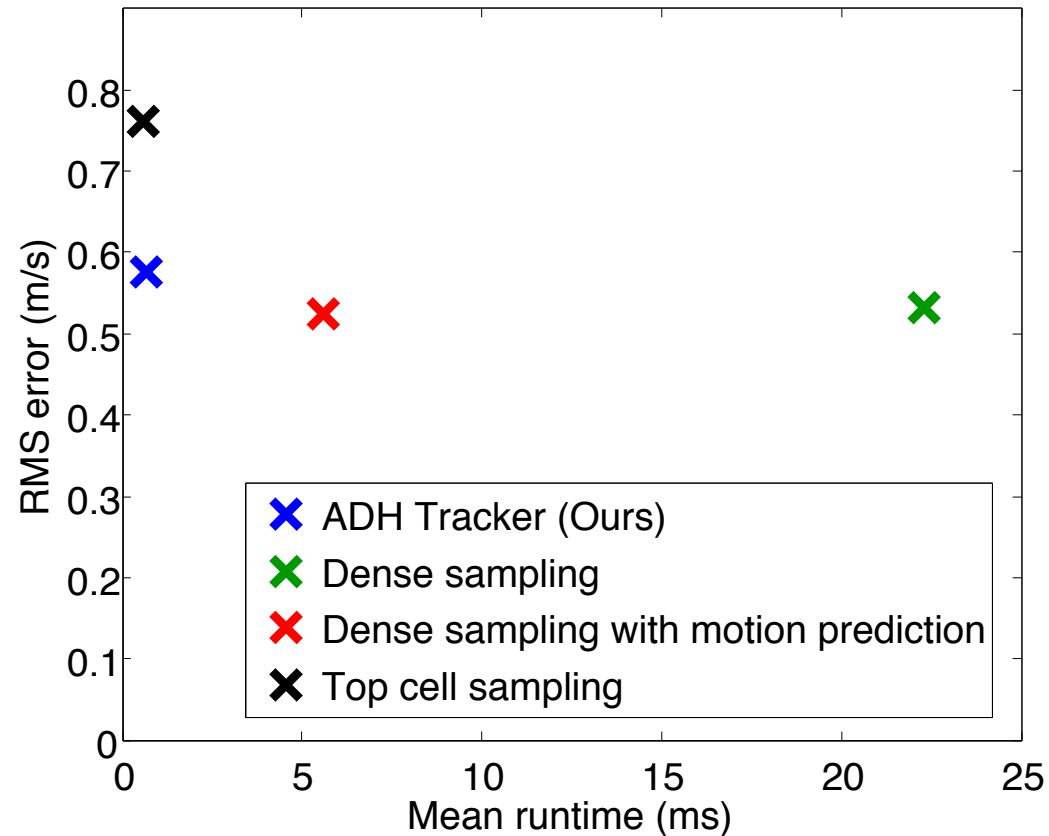
# Models



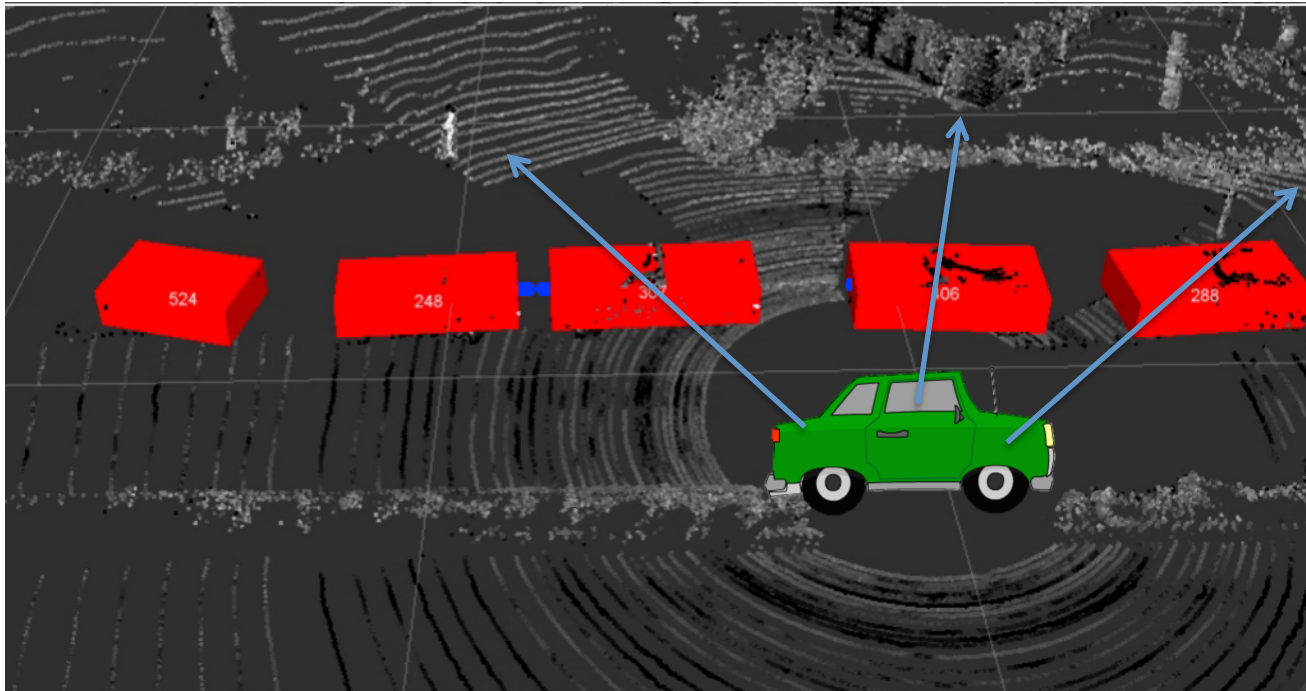
## Quantitative Evaluation 2



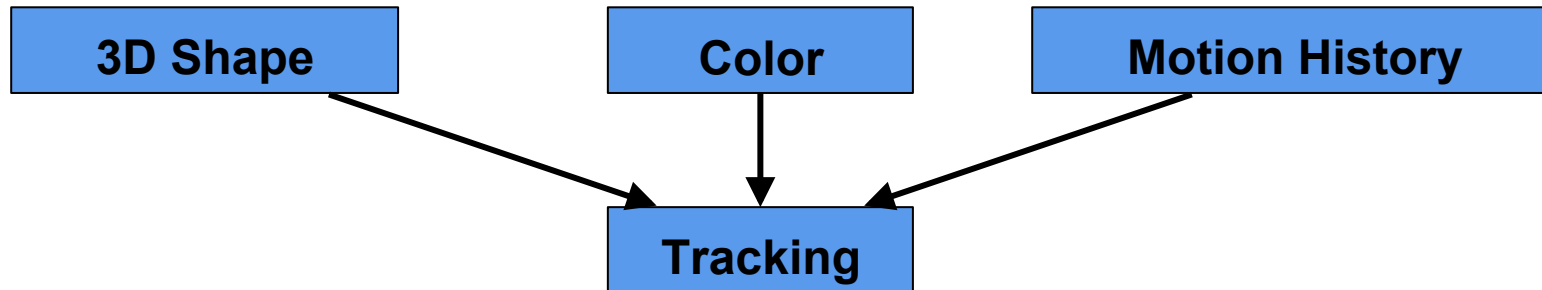
# Sampling Strategies



# Advantages over Radar

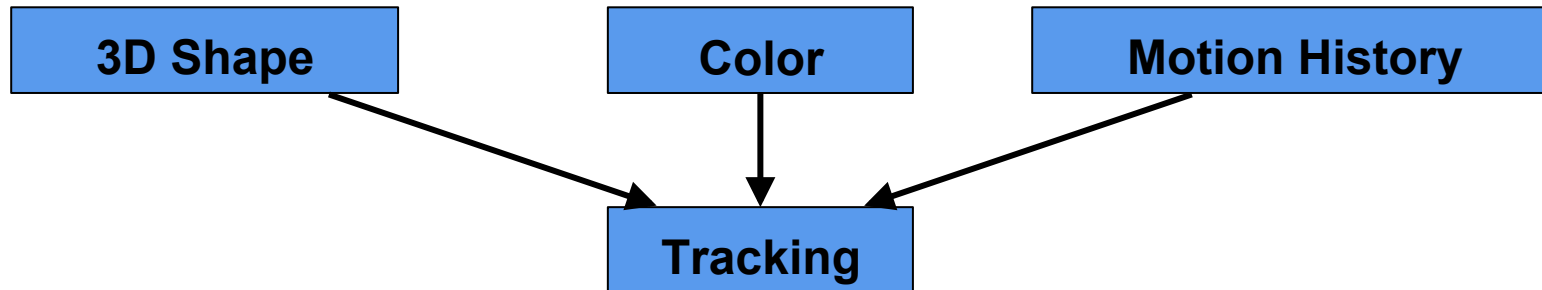


# Conclusions

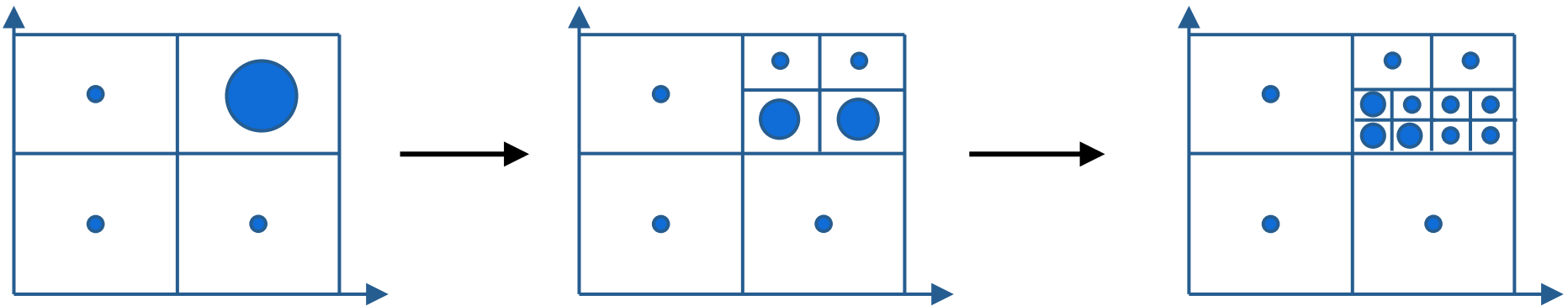


- **Robust to Occlusions, Viewpoint Changes**

# Conclusions



- **Robust to Occlusions, Viewpoint Changes**

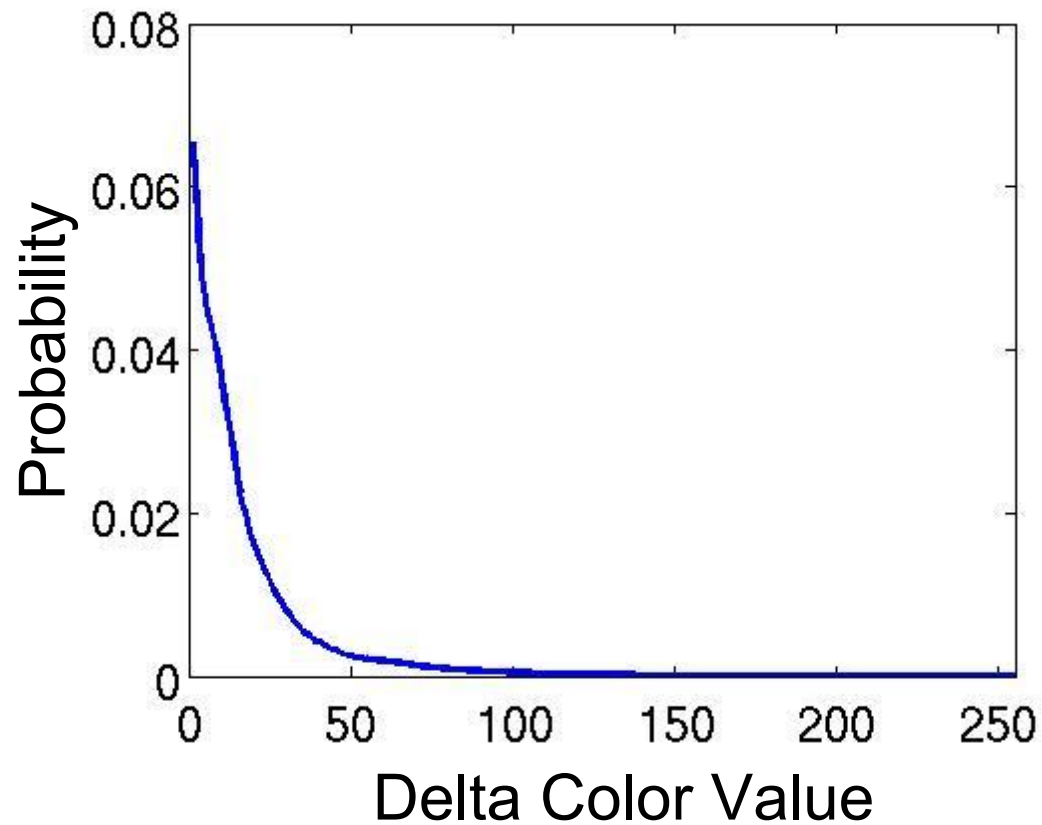


- **Runs in Real-time**
- **Robust to Initialization Errors**

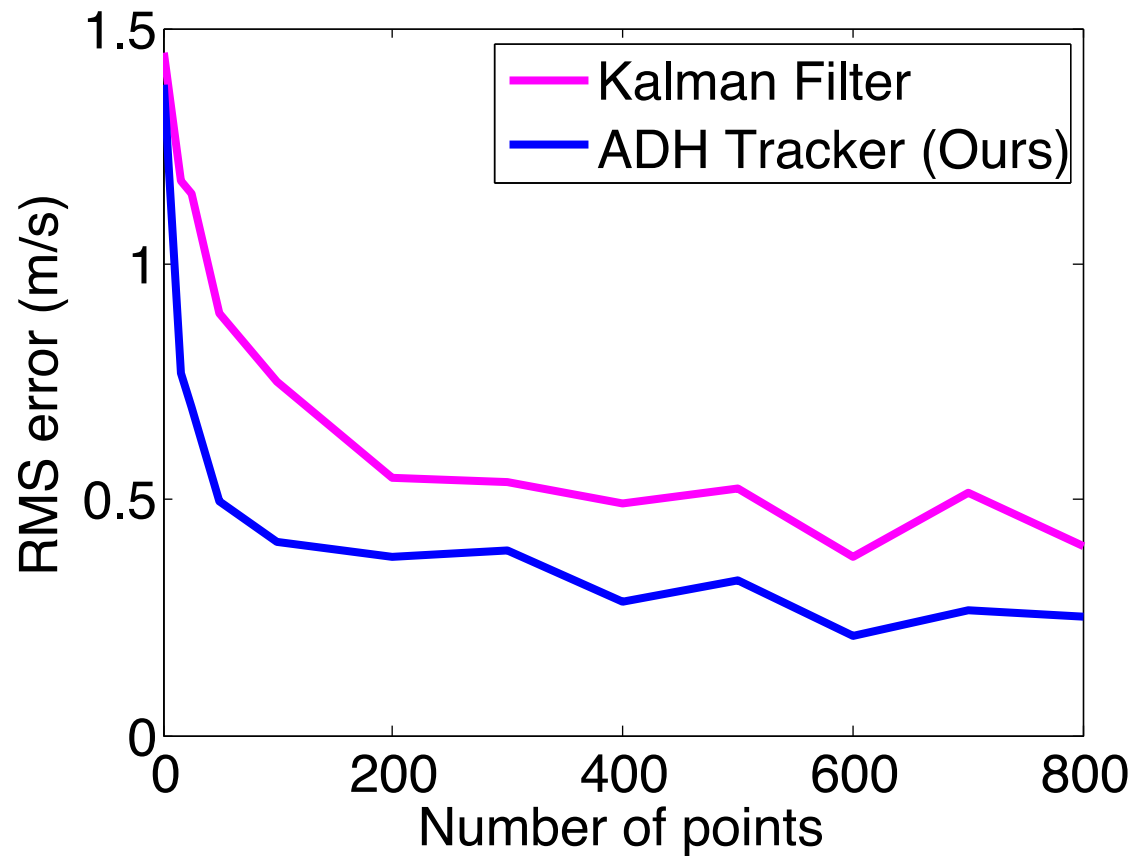




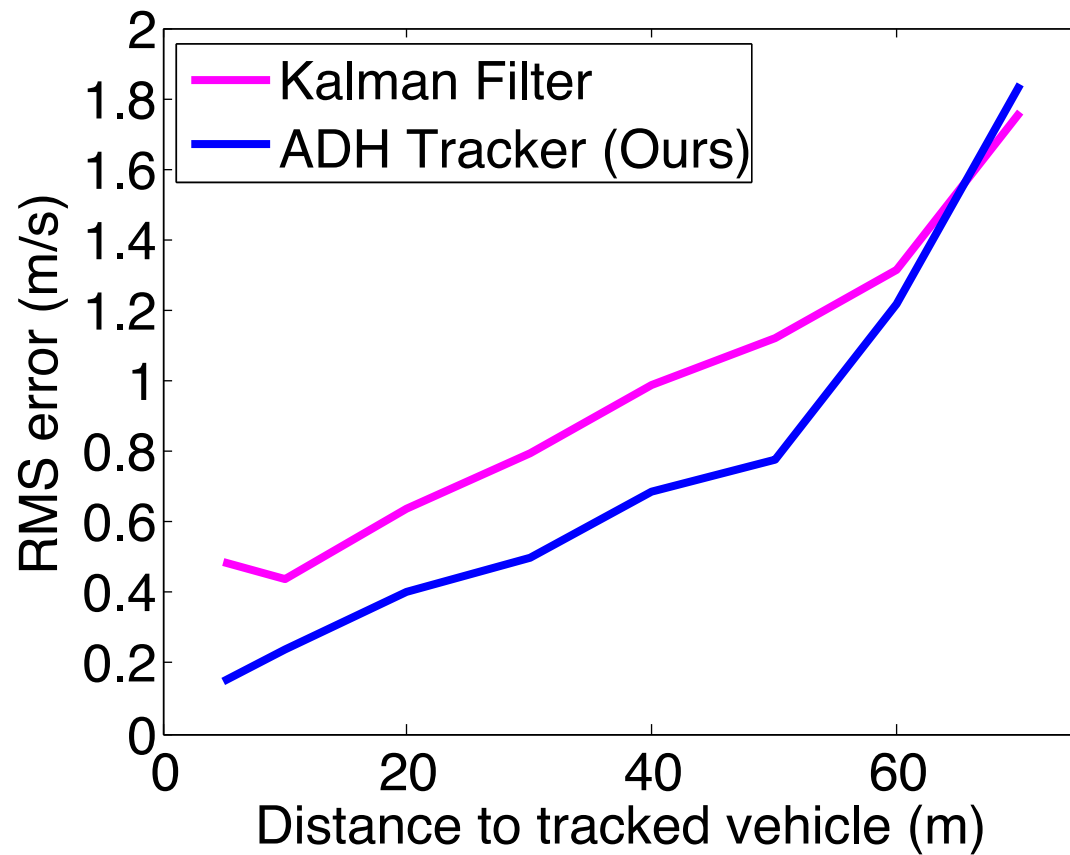
# Color Probability



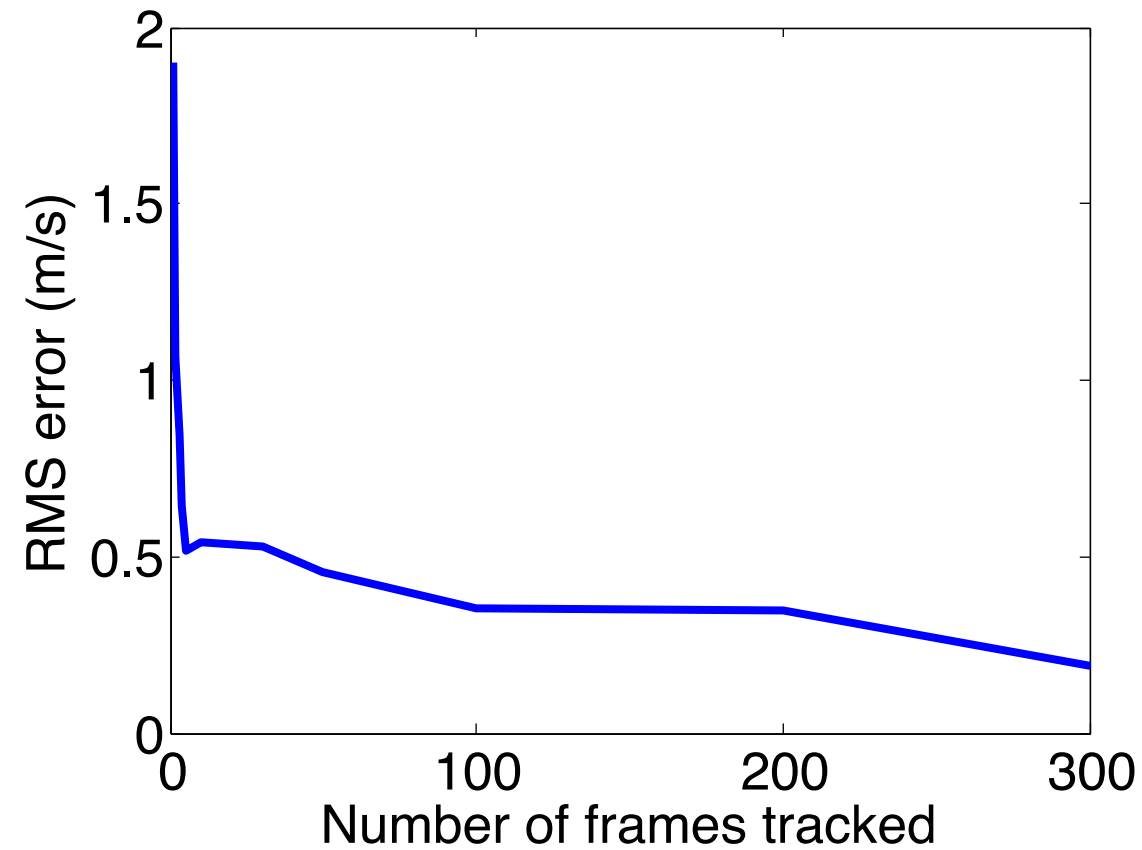
# Error vs Number of Points



# Error vs Distance



# Error vs Number of Frames



# Error vs Number of Frames

